

Sequences and Series, Functions and Inverses

REVISION TEST

120 Marks $2\frac{1}{2}$ hours

SECTION ONE

56 MARKS

Question One

1. If $S_n = \frac{3}{2}n^2 - \frac{5}{2}n$

1.1 Determine the sum of the first 10 terms. (2)

1.2 Determine T_{10} (3)

(5)

Question Two

2. Three numbers form an arithmetic sequence. Their sum is 24.

2.1 If a is the first number and d the common difference, show that $a + d = 8$. (2)

2.2 If the first number is decreased by 1 and the second number by 2, the three numbers then form a geometric sequence. Find the three numbers. (8)

(10)

Question Three

3. Consider the number pattern 6; 10; 16; 24; 34;

3.1 If the pattern behaves consistently, determine the next TWO terms of the pattern. (2)

3.2 Determine the general term of this pattern. (5)

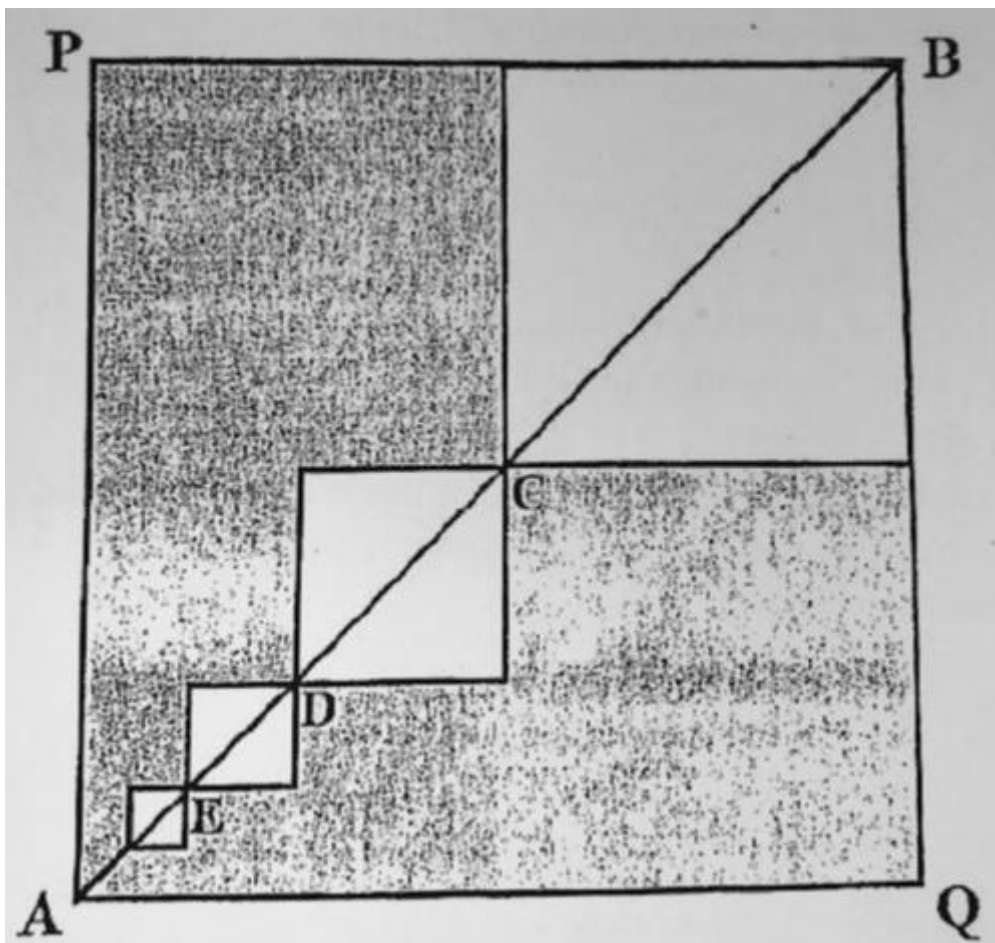
3.3 Calculate n if the n^{th} term in the pattern is 1264. (4)

(11)

Question Four

The diagram below represents a square $PBQA$ with side lengths equal to 12 units. A second square is drawn with diagonal $CB = \frac{1}{2}AB$. A third square is drawn with diagonal

$DC = \frac{1}{2}AC$, and a fourth square with diagonal $ED = \frac{1}{2}AD$. The pattern continues until 10 Squares are drawn.



Determine the area of the shaded section.

(5)

(5)

Question 5

5.1 Given the series $(2x + 1) + (2x + 1)^2 + (2x + 1)^3 + (2x + 1)^4 \dots$

5.1.1 Determine the values of x for which the series will converge. (3)

5.1.2 Determine S_{∞} if $x = -\frac{1}{4}$ (3)

5.2 Calculate the value of n if

$$\sum_{k=1}^n \frac{1}{4} (2)^{k-1} = \frac{511}{4}$$

(4)

5.3 Evaluate

$$\sum_{r=0}^{\infty} 2 \left(\frac{1}{2}\right)^r - \sum_{r=1}^5 2$$

(4)

(14)

Question Six

A quadratic Sequence has the following properties:

$$T_{10} = T_{14} = 0$$

$$T_{17} = -21$$

6.1 Which term will have the largest value in the pattern? (1)

6.2 Which other term will have a value of -21? (1)

6.3 Show $T_n = -n^2 + 24n - 140$ (4)

6.4 If F_n is the first difference of this quadratic pattern:

6.4.1 Determine an expression for F_n (3)

6.4.2 What is the difference between the 91st and 92nd term of the quadratic pattern? (2)

(11)

Question One

Consider the function $g(x) = \log_{\frac{1}{2}} x$

1.1 Will the graph of g be increasing or decreasing? Motivate your answer. (2)

1.2 Find the equation of $g^{-1}(x)$ and draw a neat sketch of both $g(x)$ and $g^{-1}(x)$ on the same set of axes. (6)

1.3 Write down the equation of the asymptote of $g(x + 2)$. (1)

1.4 Write down the domain of $g^{-1}(x)$ (1)

(10)**Question Two**

2.1 Given $f(x) = \sqrt{4x}$ and $g(x) = x^2$, evaluate $f(g(9))$ (3)

2.2 Rewrite $f(x) = \frac{x-1}{x-2}$ in the form $f(x) = \frac{a}{x-p} + q$ and hence give the equations of the asymptotes. (5)

2.3 Given $f(x) = 3x^2 - 7$

The graph of f is shifted 3 units down and 2 units to the left, resulting in the graph of $h(x)$. Determine an expression for $h(x)$ in the form $ax^2 + bx + c$ (4)

2.4 Given $f(x) = 1 + 2^x$

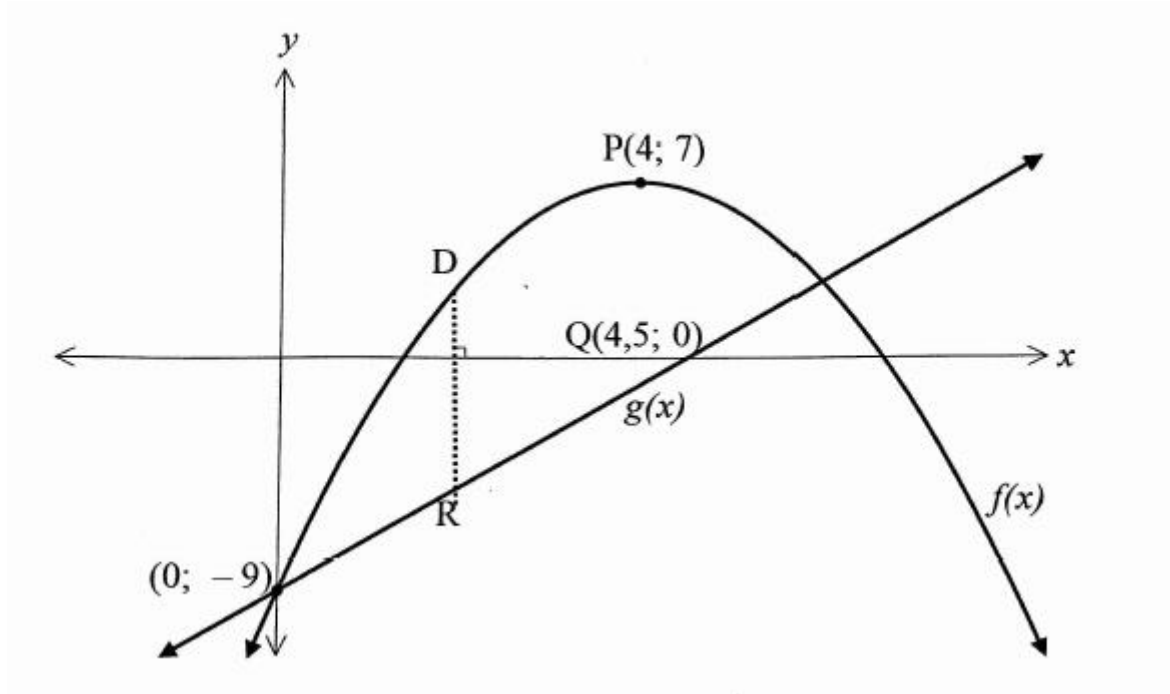
2.4.1 Show that $f(x) \times f(-x) = f(x) + f(-x)$ (3)

2.4.2 If $g(x) = f(x) - 1$, determine $g^{-1}(x)$ in the form $y = \dots$ (3)

(18)

Question Three

The diagram shows the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = 2x - 9$. P is the turning point of the parabola. Both $f(x)$ and $g(x)$ pass through the point $(0; -9)$. $g(x)$ passes through $Q(4,5; 0)$.

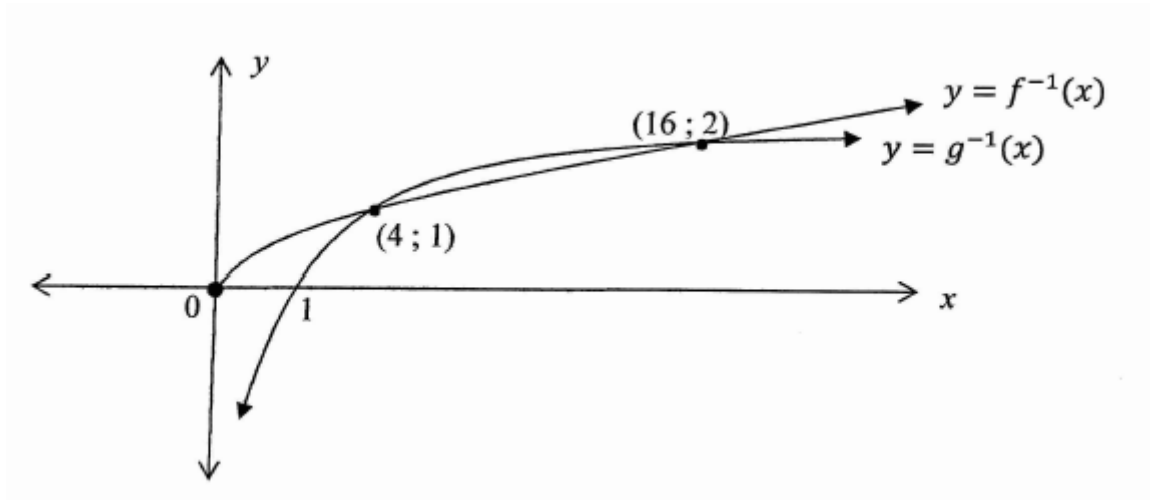


- 3.1 Write down the equation of the axis of symmetry of f . (1)
- 3.2 Write down the coordinates of the point which is a reflection of the point $(0; -9)$ in the axis of symmetry of f . (2)
- 3.3 Determine the values of a, b and c . (5)
- 3.4 Determine the length of **DR** in terms of x if D is on f and R is on g and DR is parallel to the line $x = 0$. (2)
- 3.5 Determine the value(s) of x for which DR is maximum. (2)

(12)

Question Four

The sketch below represents the inverses of $g(x) = 4^x$ and $f(x) = ax^2; x \geq 0$.



- 4.1 Write down the coordinates of ONE point through which both f and g will pass. (1)
- 4.2 Determine the equation of f . (3)
- 4.3 Calculate x if $g(x + 2) = 16$. (3)
- 4.4 If $h(x) = g^{-1}(x - 2)$, for which values of x will $h(x) \leq 0$? (2)

(9)

Question 5

5.1 Given $g(x) = 2x^2 - 3x - 2$:

5.1.1 Determine a, p and q such that $g(x) = a(x - p)^2 + q$ (3)

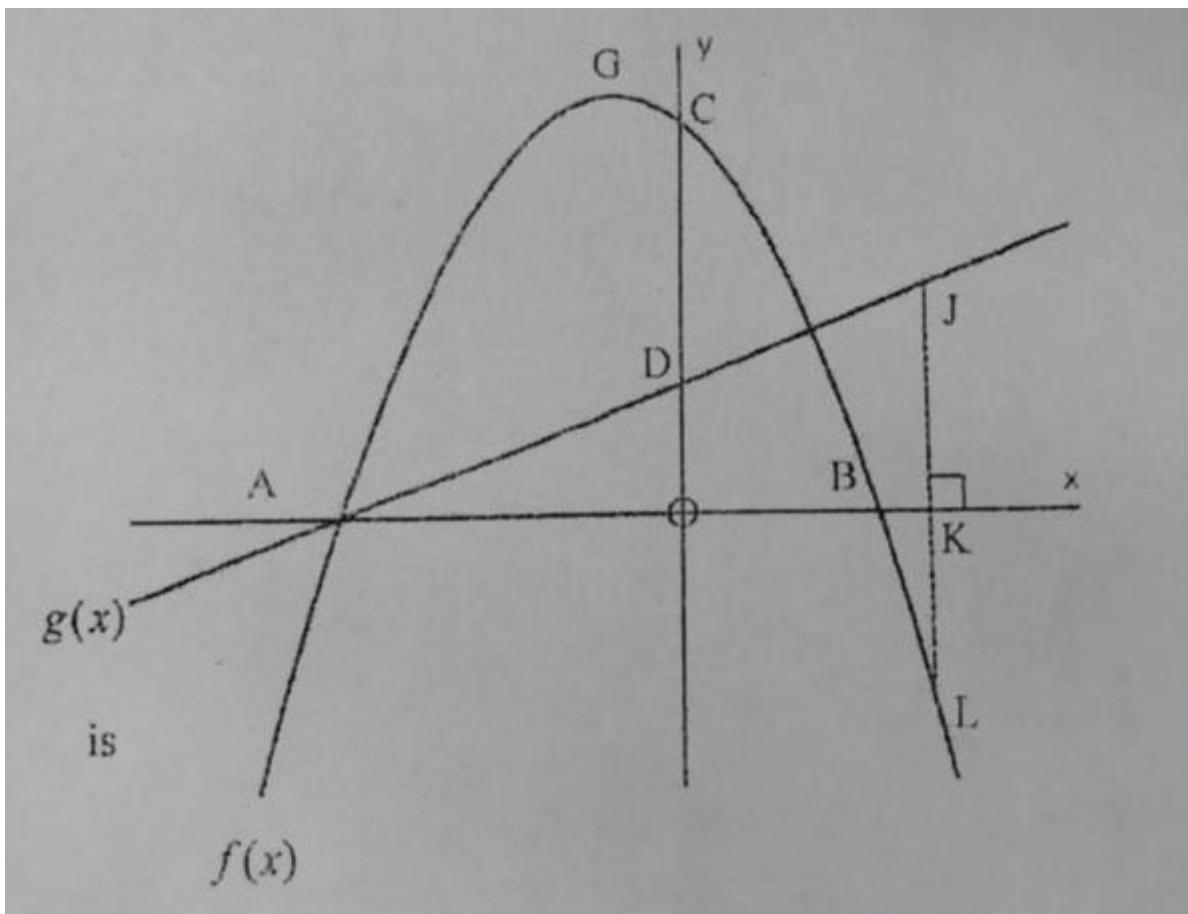
5.1.2 Write down the coordinates of the turning point of g . (2)

5.1.3 For which values of $p, p \in R$ will the equation $g(x) = p$ have real roots? (2)

5.2 Refer to the figure below.

The graphs of $f(x) = -2x^2 + bx + 30$ and $g(x) = 2x + 10$ are drawn.

A and B are the x -intercepts and C is the y -intercept of $f(x)$. G is the turning point of $f(x)$. A is the x -intercept and D is the y -intercept of $g(x)$.



5.2.1 Show, by performing the necessary calculations, that $b = -4$ (4)

5.2.2 If $JL = 60$ units, determine the length OK , where K is the point on the x -axis, defined by the vertical line JL . (4)

(15)