

**Problem 1**

Assume that there are one red ball and three black balls, and the balls are lined up randomly. Define the value of random variable for k-th ball ( $k=1, 2, 3, 4$ ) as follows:

$$X_k = \begin{cases} 0 & \text{if the color of the k-th ball is black} \\ 1 & \text{if the color of the k-th ball is red.} \end{cases}$$

(1) Show the joint probability function of  $X_1, X_2, X_3, X_4$  is the same.

Example : If  $\bullet \bullet \bullet \bullet$ , we get  $(X_1, X_2, X_3, X_4) = (0, 1, 0, 0)$

(2) Explain that the probability of winning one Amidakuji is irrelevant to the order in which you choose. (Amidakuji is also called a ladder lottery.)

**Problem 2**

In a sequence of characters “a” and “b” randomly arranged, let X be the number of characters until the sequence “ab” first appears, and let Y be the number of characters until the sequence “aa” first appears. What is the magnitude relationship between the expected values of the numbers of X and Y ?