

Question 1

- (a) A given number in base x can be converted to any other base y . According to the expansion method, if **abc.de** is any given number in base x , then write its value in base 10. [3]
- (b) Convert the following numbers using number system conversions, show your answer in details: [6]
- i. $(723)_8$ to hexa decimal system
 - ii. $(0.ABDF)_{16}$ to decimal system
 - iii. Convert 0.375 to binary system
 - iv. Which digits from (0,1,2,3,4,5) are not allowed in Quinary system (base 5) representation.
 - v. $(11010.1011)_2$ to hexadecimal.
 - vi. $(257)_{10}$ to the binary system.
- (c) Consider the binary number 10.0011 [4]
- i. Convert the above number to the decimal system
 - ii. What are the place values of the digits 1 in the number 0.0011_2
 - iii. what is the sum of (1+1+1+1) in binary system
 - iv. calculate 101 divided by 10 using long division.
- (d) Which one is the correct representation of a binary number from the following? [2]
- i. 1101
 - ii. $(214)_2$
 - iii. $(0000)_2$
 - iv. $(11)^2$

Question 2

- (a) Is $a_n = \frac{3n+2}{n-4}$ a general term of a sequence? Why? [2]
- (b) Which term of the sequence with general term $\frac{3n-1}{5n+7}$ is $\frac{7}{12}$? [2]
- (c) An arithmetic sequence has its 4^{th} term equal to 18 and its 12^{th} term equal to 50. Find its 99^{th} term. [4]



- (d) State whether the following sequences are arithmetic, geometric or not any of them. Find the common ratio if it is a geometric sequence and find the common difference d if it is an arithmetic sequence. Then, find the next two terms. [6]
- $-3, 3, -3, 3$
 - $b_n = n^2 + 3$
 - $\frac{-1}{2}, \frac{-5}{6}, \frac{-7}{6}$
- (e) Consider the geometric sequence (b_n) with $b_1 = \frac{1}{9}$ and $q = 3$. Is 243 a term of this sequence? [3]
- (f) The nineteenth term of a sequence is -52, and the fourth term is -7. The difference between consecutive terms in the sequence is constant. Find the 201st term. [3]
- (g) Show whether the following sequence is convergent or divergent. [2]
 $\lim_{n \rightarrow \infty} \left(\frac{n-1}{n} \right)$
- (h) Is the following numbers 1,-4,9,-16,... represent a sequence, if so, find a formula for the n^{th} term of the sequence. [2]
- (i) Show by mathematical induction that for all positive integers n ,
 $\frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$ [4]
- (j) Find the remainder when 3^{123} is divided by 7. [2]

Question 3

- (a) State whether the following statements are false or true, explain your answer: [6]
- Given any integers a, b, c and any positive integer n
 If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.
 - Suppose $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then
 $a + c \equiv b + d \pmod{n}$.
 - $7x \equiv 12 \pmod{7}$.
- (b) Find the least positive value of x such that: [3]
 $71 \equiv x \pmod{8}$
- (c) Calculate the multiplicative inverse of 168 in modulo 83. [3]



- (d) Calculate the inverse of 4 modulo 15. Show your steps. [3]

Question 4

- (a) A triangle has sides $a = 2$ and $b = 3$ and angle $C = 60^\circ$. Find [4]
- the length of side c .
 - Find the sine of angle B using sine rules.
- (b) If we have a triangle which has one of its side $c = 2$ and angles $A = \pi/4$ and $B = \pi/3$. Work out the length a of the side opposite A . [4]
- (c) XYZ is a right angled triangle with $Y = 90^\circ$. Given that $y = 85$, $\sin X = \frac{77}{85}$, find z , $\cos(Z)$ and the angle of Z . [6]
- (d) Let g be a function with its domain $(0, \infty)$, defined by $g(x) = \frac{1}{x}$. [6]
- Sketch the graph of g .
 - Is g continuous at other points of its domain?

Question 5

- (a) Consider the function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ given by

$$f(x) = \begin{cases} n + 1, & \text{if } n \text{ is even} \\ n - 3, & \text{if } n \text{ is odd} \end{cases}$$

[6]

- Is f injective? Prove your answer
 - Is f surjective? Prove your answer
- (b) The function $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined as follows:

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x^2, & \text{if } 0 \leq x \leq 1. \\ 1, & \text{if } x > 1 \end{cases}$$

Plot the function, and say whether it is Bijective function or not. Explain your answer. [5]



- (c) The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

Find the average acceleration in the time interval $t = 0$ to $t = 2$ sec. [4]

- (d) An object moving with uniform acceleration has a velocity of 12 cm/s in the positive x direction when its x coordinate is 3.00 cm . If its x coordinate 2 seconds later is -5 cm , what is its acceleration? [5]

END OF PAPER

