

## **Engineering**

**Mathematics and Statistics for Engineers**

**IT Skills coursework**

**Summative Coursework**

**Assignment 2020 – 21**

**Summer Term 2**

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### **Overview**

You need to prepare a report based on the tasks that you can find below. You need to include appropriate images of your work done using GeoGebra to attach as a proof of your answers. Pictures of written work will not be marked. You need to use the Word equation editor and the GeoGebra software.

**Assessment Details:**

<b>Hand out Date:</b>	08/07/2021	<b>Submission Date:</b>	15/07/2021	<b>Assessment Duration:</b>	8 days
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**Notes:**

This assessment consists of **66** marks and will comprise **10%** of your overall mark for the module.

**Knowledge and Understanding**

- 2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;
- 3) evaluate the strengths and weaknesses of alternative models and consequently of any conclusions drawn;

**Subject-specific Skills**

- 5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;
- 7) employ a range of numeric techniques to find approximate solutions to problems;

**Key and Employability Skills**

- 8) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;

**Please note:** You are reminded that plagiarism is the uncited use of other's work – this includes graphs, diagrams, images and written text. It is therefore important to reference all source material used in your essay. If you are found to have plagiarised work you are guilty of Academic Impropriety and are likely to have your grade for that component of the assessment reduced to zero.

**Evidence for Assessment:**

- Word document containing your report
- Images exported from GeoGebra

### **Submission Details:**

You need to upload the assignment via Turnitin.

The Turnitin link can be found in the 'Assessment Tab' on your Study Smart course page.

Your report should be formatted in the following way:

- **Student ID on first page (NOT your name)**
- **Use of the equation environment in Word when necessary**

**Use of images from GeoGebra to add to your Word document**

### **Academic Impropriety:**

You are reminded that plagiarism is the unreferenced use of other people's work or your own previous work. This could include visual images, sound recordings, diagrams, as well as written text. You can however use other people's work as examples, supporting evidence or inspiration as long as it is referenced appropriately.

Academic Impropriety also includes copying or using other people's work and presenting this as your own. This could include work produced by family members, friends or unknown people on the Internet.

If you are guilty of Academic Impropriety you are likely to have your grade for the assignment reduced to zero.

Task 1 – Cubics (27 marks)	Learning Outcome
<p><b>Part A</b></p> <p>A <b>monic</b> cubic <math>P(x)</math> is such that</p> <ul style="list-style-type: none"> <li>• it has two turning points, and</li> <li>• the maximum difference between two of its roots is <math>2k</math>.</li> </ul> <p>The vertical distance between the turning points of <math>P(x)</math> is <math>d</math>.</p> <p>Use GeoGebra to investigate cubics of this type to find an expression for the minimum and maximum values of <math>d</math> and the relationship between all three roots in each case.</p> <p><b>Part B</b></p> <p>Another <b>monic</b> cubic <math>Q(x)</math> is such that it has</p> <ul style="list-style-type: none"> <li>• two turning points, and</li> <li>• rotational symmetry of <math>180^\circ</math> about the origin.</li> </ul> <p>Let <math>l</math> be a straight-line intersecting <math>Q(x)</math> at three distinct points.</p> <p>Use GeoGebra to investigate cubics of this type to find any relationship between the <math>x</math> coordinates of the intersection points between <math>l</math> and <math>Q(x)</math>.</p>	<p>2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;</p> <p>5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;</p> <p>8) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;</p>

Task 2 – Geometry (12 marks)	Learning Outcome
<p>A garden is in the shape of a rectangle with a semicircle entirely along one edge. The length of the fence around the garden is <math>P</math> metres. The corresponding <math>r</math> is the radius of the semicircle that maximises the area of the garden.</p> <ol style="list-style-type: none"> <li>Use GeoGebra to find the corresponding <math>r</math> for a range of different perimeters, <math>P</math>.</li> <li>Use GeoGebra to show that the points <math>(P, r)</math> corresponding to your values all lie, subject to rounding errors, on a straight line and find the equation of this line.</li> <li>Use your equation to find the lengths of the sides of the garden which maximises its area when the perimeter is 130 metres. Use GeoGebra to show your result in this case.</li> </ol>	<ol style="list-style-type: none"> <li>identify whether a real-life situation is best solved exactly, numerically, or probabilistically;</li> <li>choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;</li> <li>employ a range of numeric techniques to find approximate solutions to problems;</li> <li>use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;</li> </ol>

Task 3 - Calculus (12 marks)	Learning Outcome
<p>You are given that any reasonable function <math>f(x)</math> can be approximated by a Taylor Series about <math>x = 0</math> of appropriate order, with the formulas:-</p> <p><b>Order 1 (linear)</b></p> $f(x) \approx f(0) + f'(0)x$ <p><b>Order 2 (quadratic)</b></p> $f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$ <p><b>Order 3 (cubic)</b></p> $f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$ <p><b>and similarly for higher orders.</b></p> <ol style="list-style-type: none"> <li>For the function <math>f(x) = \frac{e^{2x}}{\sqrt{x^2+1}}</math>, use GeoGebra to find the value at <math>x = 0</math> of <math>f(x)</math> and up to the 5th derivative for the function</li> <li>Use the values found in part a. to write down the equations of the linear, cubic and quintic approximations to the function.</li> <li>Use GeoGebra to plot the functions found in part b and the Integral function in GeoGebra to find the approximate area under each of the approximations in part b. between <math>x = 0</math> and <math>x = 1</math>.</li> <li>Also use the Integral function in GeoGebra to find the area under the original curve over the same domain and comment on the accuracy of the approximations.</li> </ol>	<p>2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;</p> <p>3) evaluate the strengths and weaknesses of alternative models and consequently of any conclusions drawn;</p> <p>5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;</p> <p>8) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;</p>

Task 4 – Numerical Integration (15 marks)	Learning Outcome
<p>The integral</p> $\int_a^b f(x) dx$ <p>can be approximated by modelling <math>f(x)</math> by a piecewise linear or quadratic function respectively.</p> <p>You are asked to investigate the area under the curve with definition</p> $f(x) = 25x^2e^{-3x}$ <p>between <math>x = 1</math> and <math>x = 2</math>.</p> <p><b>Part A</b></p> <p>Assume first that you are using a linear model and so using the Trapezium rule to find an estimate of the area.</p> <ol style="list-style-type: none"> <li>Plot the function <math>y = f(x)</math> explain whether you expect the estimate to be an overestimate or underestimate.</li> <li>Use an appropriate spreadsheet to find the minimum number of ordinates necessary to find an estimate to the area correct to 1 decimal place.</li> </ol> <p><b>Part B</b></p> <p>Assume now that you are using a quadratic model and so using Simpson's rule to find an estimate of the area.</p> <ol style="list-style-type: none"> <li>Use an appropriate spreadsheet to find the minimum number of ordinates necessary to find an estimate to the area correct to 1 decimal place.</li> </ol> <p><b>Part C</b></p> <ol style="list-style-type: none"> <li>Compare the two models, discussing the advantages and disadvantages of both.</li> </ol>	<p>2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;</p> <p>3) evaluate the strengths and weaknesses of alternative models and consequently of any conclusions drawn;</p> <p>5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;</p> <p>7) employ a range of numeric techniques to find approximate solutions to problems;</p> <p>8) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;</p>

## Grade Rubric – Mathematics and Statistics for Computer Scientists

### IT Skills

TASK 1 Part A	LEARNING OUTCOME	ASSESSMENT CRITERIA	0	1	2	3
			Poor	Satisfactory	Good	Excellent
Using GeoGebra plot the cubics	2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;  5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;  9) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;	<b>Using IT</b>  <b>Plotting cubics</b>  <b>Drawing straight lines</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Any cubic plotted</i>	<i>A cubic plotted with the required maximum distance between the roots</i>	<i>More than one cubic plotted with the required maximum distance between the roots and different third roots shown</i>
Calculation of d		<b>Finding turning points and defining a variable</b>	<i>Little or no evidence of working in Geogebra</i>	<i>The turning points have been identified</i>	<i>A variable has been defined for the vertical difference between the turning points</i>	<i>More than one example shown where d is an extreme</i>
Relationship when d is a maximum		<b>Interpreting extremes</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Situation identified when d is a maximum</i>	<i>Explanation of when the maximum happens or attempt to find expression for d</i>	<i>Correct expression for d found and correct explanation of when this occurs</i>
Relationship when d is a minimum		<b>Interpreting extremes</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Situation identified when d is a minimum</i>	<i>Explanation of when the minimum happens or attempt to find expression for d</i>	<i>Correct expression for d found and correct explanation of when this occurs</i>



TASK 1 – Part B	LEARNING OUTCOME	ASSESSMENT CRITERIA	0	1	2	3
			Poor	Satisfactory	Good	Excellent
Using GeoGebra plot the cubic	2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;  5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;  9) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;	Using IT  Plotting cubics	<i>Little or no evidence of working in Geogebra</i>	<i>Any cubic plotted</i>	<i>A cubic plotted with the required number of turning points</i>	<i>A cubic plotted with appropriate symmetry and number of turning points</i>
Drawing a line		Defining two points and a line	<i>Little or no evidence of working in Geogebra</i>	<i>Two points plotted on the cubic</i>	<i>Two points plotted on cubic and line joining them</i>	<i>Two points plotted, line joining them and third intersection point</i>
Finding a minimum area		Finding area	<i>Little or no evidence of working in Geogebra</i>	<i>Any attempt to find an area between the line and the curve</i>	<i>An integral is calculated for the finite regions</i>	<i>A correct variable is defined finding the area is found</i>
Finding minimum areas		Minimum areas	<i>Little or no evidence of working in Geogebra</i>	<i>Any attempt to vary a point to find the minimum area</i>	<i>Correctly identifies the minimum area for a given fixed point</i>	<i>Correctly identifies the minimum area for several different fixed points</i>
Relationship when a minimum		Identifies the relationship between the coordinates	<i>Little or no evidence of working in Geogebra</i>	<i>Attempts to interpret their results</i>	<i>Identifies any relationship for their results</i>	<i>Correctly identifies the relationship between the x-coordinates</i>

TASK 2	LEARNING OUTCOME	ASSESSMENT CRITERIA	0	1	2	3
			Poor	Satisfactory	Good	Excellent
Plots a plan of the garden	2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;  5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;	<b>Plotting shapes</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Any attempt to plot a plan for the garden of the right shape</i>	<i>Plots a plan of the garden of the right shape for a specified perimeter</i>	<i>Plots a plan of the garden of the right shape for a perimeter based on a slider</i>
Finding values	7) employ a range of numeric techniques to find approximate solutions to problems;	<b>Finding r by changing slider</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Correctly identifies a value for r for a specified perimeter</i>	<i>Correctly identifies values for r for more than one perimeter</i>	<i>Correctly identifies values for r for a range of perimeters</i>
Plotting points and show fits on a line	9) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;	<b>Plotting points and best fit line</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Attempt to plot their points</i>	<i>Points plotted correctly that are very close to a straight line</i>	<i>Points plotted and equation of best fit line found</i>
Find prediction and show correct	analysis and solution of problems;	<b>Prediction and verification</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Attempt to find value of r from equation</i>	<i>Correctly finds value of r</i>	<i>Correctly finds value of r and demonstrates with example on geogebra</i>

TASK 3	LEARNING OUTCOME	ASSESSMENT CRITERIA	0	1	2	3
			Poor	Satisfactory	Good	Excellent
Finding values of $f$ and its derivatives.	2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;	<b>Deriving and evaluating derivatives</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Any attempt to find the value of <math>f</math> or a derivative at 0</i>	<i>Value of <math>f</math> and at least one derivative found correctly at 0</i>	<i>All 6 values found correctly</i>
Writing down the equations	3) evaluate the strengths and weaknesses of alternative models and consequently of any conclusions drawn;	<b>Finding Taylor Series</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Attempt to find one of the Taylor series, most likely the linear</i>	<i>Correctly finds at least 2 of the specified Taylor series based on their values</i>	<i>Correctly finds all three Taylor series</i>
Plotting functions and finding area.	5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;	<b>Plotting and integrating to find areas</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Any attempt to plot one of the approximations and find an area</i>	<i>At least one approximation plotted correctly and the area found</i>	<i>All three approximations plotted correctly and the areas found</i>
Correct area found and comments on accuracy	9) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;	<b>Finding an area and commenting</b>	<i>Little or no evidence of working in Geogebra</i>	<i>Attempt to find the correct area</i>	<i>Correct area found and any appropriate comment made</i>	<i>Correct area found and appropriate comparisons about accuracy made</i>

TASK 4	LEARNING OUTCOME	ASSESSMENT CRITERIA	0	1	2	3
			Poor	Satisfactory	Good	Excellent
Plot curves and discuss overestimate or underestimate	2) identify whether a real-life situation is best solved exactly, numerically, or probabilistically;  3) evaluate the strengths and weaknesses of alternative models and consequently of any conclusions drawn;  5) choose and apply appropriate techniques in algebra, co-ordinate geometry, trigonometry and calculus in finding a solution to a problem;  9) use a range of IT packages, including calculators, spreadsheets, word processors, and skills in the analysis and solution of problems;	Plotting curve and discussing estimate	<i>Little or no evidence</i>	<i>Curve plotted correctly</i>	<i>Curve plotted correctly and statement of whether an underestimate or overestimate</i>	<i>Curve plotted correctly and statement of whether an underestimate or overestimate with valid reason</i>
Trapezium Rule		Find data for trapezium rule	<i>Little or no evidence</i>	<i>Any attempt to identify h and ordinates</i>	<i>Attempts to find an h, ordinates and y values</i>	<i>Correctly finds an h, ordinates and y values based on a specified number of ordinates</i>
Trapezium Rule		Finding area	<i>Little or no evidence</i>	<i>Any attempt to find area using a spreadsheet and trapezium rule</i>	<i>Correctly finds an area using the trapezium rule</i>	<i>Correctly finds the minimum number of ordinates to find the area to the correct accuracy.</i>
Simpson's Rule		Finding an area	<i>Little or no evidence</i>	<i>Any attempt to find area using a spreadsheet and Simpson's rule</i>	<i>Correctly finds an area using Simpson's Rule</i>	<i>Correctly finds the minimum number of ordinates to find the area to the correct accuracy.</i>
Comparison		Discussion of models	<i>Little or no evidence</i>	<i>Any discussion of the results using one rules</i>	<i>Any comparison of the results from the two rules</i>	<i>A valid comparison between the results</i>