MICROCOMPUTERS IN POLICY RESEARCH

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4a

EXERCISES IN GENERAL EQUILIBRIUM MODELING USING GAMS

HANS LÖFGREN

INTERNATIONAL FOOD POLICY RESEARCH INSTITUTE

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PREFACE

Over the past decade, the increasing power and reliability of microcomputers and the development of sophisticated software designed specifically for use with them has led to significant changes in the way quantitative food policy analysis is conducted. These changes cover most aspects of the analysis, ranging from the collection and analysis of socioeconomic data to the conduct of model-based policy simulations. The venue of the computations has shifted from off-site mainframes dependent on highly trained operators and significant capital investment in supporting equipment, to desktop and laptop computers, dependent only on the occasional availability of electricity. This means that it is now feasible to quickly transfer new techniques between IFPRI and IFPRI's collaborators in developing countries, that the costs of policy analysis have been substantially reduced, and that a new level of complexity and accuracy in policy analysis is now possible.

As with any new technology, however, there are substantial costs in time and money involved in learning the most efficient ways of using this new technology and then transmitting these lessons to others. This series, Microcomputers in Policy Research, represents IFPRI's collective ongoing experience in adapting microcomputer technology for use in food policy analysis in developing countries. The papers in the series are primarily for the purpose of sharing these lessons with potential users in developing countries, although persons and institutions in developed countries may also find them useful. The series is designed to provide hands-on methods for quantitative food policy analysis. In our opinion, examples provide the best and clearest form of instruction; therefore, examples—including actual software codes wherever relevant—are used extensively throughout this series.

This fourth book in the series, *Exercises in General Equilibrium Modeling Using GAMS* by Hans Löfgren of IFPRI, presents a set of exercises relating to computable general equilibrium (CGE) models. CGE models represent one type of economywide model used in policy analysis. This type of model explicitly recognizes that changes that affect one part of the economy can have repercussions throughout the economy. The model is particularly useful in capturing the indirect effects of a policy change. The exercises in this book were developed for use in a master's level course in CGE modeling taught by the author while at the American University of Cairo, and they have been further refined through the recent work of IFPRI's Trade and Macroeconomics Division. The purpose of the book is to develop the ability of the reader to construct, modify, and conduct food policy simulations with CGE models using the GAMS language. The book comes with a CD-ROM with a limited-capacity version of GAMS.

Howarth Bouis and Lawrence Haddad, Series Editors

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INTRODUCTION AND OVERVIEW

Computable General Equilibrium (CGE) models are a class of economy-wide models widely used in policy analysis. The term “computable” refers to the fact that the model solution can be computed, a prerequisite when a model is used for applied purposes. A general equilibrium model explicitly recognizes that an exogenous change (in policy or from some other source, for example world markets) that affects any one part of the economy can produce repercussions throughout the system. General equilibrium models are preferable to partial equilibrium models for understanding the impact of exogenous shocks. Mathematically, a standard CGE model consists of a set of simultaneous nonlinear equations. Economically, its starting point is Walras’ neoclassical world. However, CGE models used for applied policy analysis, including food policy, tend to deviate considerably from this starting point, incorporating a relatively large amount of detailed real-world structure.

This manual presents a revised version of a set of five exercises initially developed for use in a master’s level course in CGE modeling taught by the author while at the American University in Cairo. Earlier versions have also been used in the graduate program in economics at George Washington University, Washington, D.C., and in training programs for researchers held in Tunisia (at l’Institut d’Economie Quantitative, Ministère du Développement Economique) and Malawi (at Bunda College of Agriculture, University of Malawi). The materials are also appropriate for advanced undergraduate courses.

The purpose of the exercises is to develop the ability of the reader to construct, modify, and conduct simulations with CGE models in the GAMS (General Algebraic Modeling System) language, the format of which is closely linked to standard mathematical notation.[1] This manual comes with a CD-ROM with a limited-capacity version of GAMS, including solvers for linear, nonlinear, and mixed-complementarity problems. The approximate background requirements are computer literacy, basic familiarity with GAMS, knowledge of macro and microeconomic theory at the intermediate level, and basic mathematics for economists (including the ability to derive first-order conditions for constrained optimization problems). To carry out these exercises, you need

[1]GAMS consists of a language compiler and integrated solvers. GAMS has strong data processing facilities and can be used to solve a wide variety of optimization and simultaneous equation models. It is one of the most popular softwares for solving CGE models. The web site of the GAMS Development Corporation (www.gams.com) provides information on how to acquire the full-capacity versions of the software as well as access to a wide range of GAMS-related resources, including the manual (Brooke, Kendrick, Meeraus, and Raman 1998).

Table 1—Outline of exercise content

Exercise	Content/New feature
1	Simple CGE model
A1	Simple CGE model in "longhand"
2	Intermediate demands
3	Savings and investments; activity-specific wages
4	Government; labor unemployment; activity-specific capital
5	Rest of world (open economy)

a personal computer of the IBM-type, the GAMS software, and an editor that can generate ASCII files.[2] These exercises should be studied in conjunction with other materials on general equilibrium modeling and relevant economic theory.[3]

The content of the exercises is outlined in Table 1. The approach is to start with a very simple model and subsequently modify it step by step. With very few exceptions, the models of this volume are based on data presented in the form of Social Accounting Matrices (SAMs). In Exercise 1, the mathematical statement of the simple model and its SAM are provided; the reader's task is to implement the model in GAMS. In the model statements for these exercises (as well as most models in the CGE literature), the model elements (equations, parameters, and variables) are defined over sets. The appendix to Exercise 1 includes Exercise A1, in which part of the model of Exercise 1 is rewritten in "longhand," that is, without any reference to sets. The purpose is to remind the reader of what is hidden behind references to sets and elements in model statements. Beginning with Exercise 2, cumulative modifications are introduced into the model of Exercise 1. The model of the final exercise (5), which includes a critical minimum of real-

[2] The Microsoft MS-DOS Editor, included with the MS-DOS and Microsoft Windows operating systems, is adequate. Alternatively, modelers may prefer to use the Windows-based Integrated Development Environment (IDE), included with the GAMS software.

[3] Before embarking on the exercises in this manual, the reader may, for example, study Chapters 1–3 in Devarajan, Lewis, and Robinson (1994). Extensive treatments of CGE methods are found in Dervis, de Melo, and Robinson (1982), Shoven and Whalley (1992), Dixon, Parmenter, Powell, and Wilcoxen (1992), and Ginsburgh and Keyzer (1997). Informative surveys of CGE models include Decaluwe and Martens (1988), Robinson (1989), and Bandara (1991). Condon, Dahl, and Devarajan (1987) focus on the implementation of a CGE model in GAMS. References to and examples of CGE-based analyses of food policy in developing countries done at IFPRI's Trade and Macroeconomics Division are found in the relevant section of IFPRI's website (www.ifpri.org).

world features, provides a starting point for more detailed, country-specific models that can be used for policy analysis.

In Exercises 2–4, the student is provided a verbal description of changes in the model (with various hints about how to implement them), a new SAM, and any supplementary data (if needed), on the basis of which he or she is asked first to present a new mathematical statement and, subsequently, to implement the modified model in GAMS. For the more complex Exercise 5, the task is limited to implementing the new model in GAMS; the new mathematical statement is provided. The SAM numbers in all the exercises are fictitious.

A separate document, *Key to Exercises in CGE Modeling Using GAMS,* provides suggested answers for each exercise. That document comes with a diskette with help files that facilitate the task of implementing the models in GAMS and files with suggested answers to the GAMS exercises. (For an overview, see the file README.TXT on the diskette.)

CGE modeling is not a spectator sport. It is the hope of the author that, by working through these exercises, the reader can take an important step toward using CGE models as a tool for policy analysis.

EXERCISE 1

This exercise involves implementing a simple CGE model in GAMS. The model is presented below, in both verbal terms and in the form of a mathematical statement. The model presentation is followed by a SAM that includes the data needed to solve the model using "calibration." That is, on the basis of a data set for a base period, the parameters of the model are estimated in a manner that enables the model (general equilibrium) solution to precisely replicate the base-year data set. Behavioral parameters are calibrated as if the base-year economy was indeed in equilibrium. The functional forms for the various relationships embodied in this exercise have been selected so as to assure that all parameters can be derived from the accompanying SAM. (With a few exceptions, this is also true for the rest of the exercises in this manual.)

VERBAL MODEL PRESENTATION

The model assumes that producers maximize profits subject to production functions, with primary factors as arguments, while households maximize utility subject to budget constraints. Cobb-Douglas functions are used both for producer technology and the utility functions from which household consumption demands are derived. Factors are mobile across activities, available in fixed supplies, and demanded by producers at market-clearing prices (rents). On the basis of fixed shares (derived from base-year data), factor incomes are passed on in their entirety to the households, providing them with their only income. The outputs are demanded by the households at market-clearing prices.

The model satisfies Walras' law in that the set of commodity market equilibrium conditions is functionally dependent. Any one of these conditions can be dropped. The proposed model drops the equilibrium condition for the nonagricultural commodity. The model is homogeneous of degree zero in prices. To assure that only one solution exists, a price normalization equation, in this case fixing the consumer price index (CPI), has been added. After these adjustments, the model has an equal number of endogenous variables and independent equations. Given this definition of the price normalization equation, all simulated price changes can be directly interpreted as changes vis-à-vis the CPI. The model is disaggregated into two households (urban and rural), two factors (labor and capital), and two activities and associated commodities (agriculture and nonagriculture). The explicit distinction between activities and commodities facilitates model calibration, but it is not necessary for the CGE models in this manual. The distinction, however, is needed for models that deviate from a one-to-one mapping between activities and commodities, that is, for models where at least one activity produces more than one commodity and/or at least one commodity is produced by more than one activity. The label "simple" is well deserved because the model does not include a government, intermediate demands, savings, investment, or an outside world.

MATHEMATICAL MODEL STATEMENT

The mathematical statements and the GAMS input files that accompany this volume follow the current standard notation used in CGE models developed in IFPRI's Trade and Macroeconomic Division. All endogenous variables are written in uppercase Latin letters, whereas parameters (including variables with fixed or exogenous values) have lower-case Latin or Greek letters. Subscripts refer to set indexes, with one letter per index. Superscripts are part of the parameter name (that is, not an index). In terms of letter choices, variables and parameters for commodity and factor *quantities* start with the letter *q;* for commodity and factor *prices,* the first letters are *p* and *w,* respectively.[4]

Notation

Sets

$a \in A$	activities {AGR-A agricultural activity NAGR-A nonagricultural activity}
$c \in C$	commodities {AGR-C agricultural commodity NAGR-C nonagricultural commodity}
$f \in F$	factors {LAB labor CAP capital}
$h \in H$	households {U-HHD urban household R-HHD rural household}

Parameters

ad_a	efficiency parameter in the production function for activity a
cpi	consumer price index (CPI)
$cwts_c$	weight of commodity c in the CPI
$shry_{hf}$	share for household h in the income of factor f
qfs_f	supply of factor f
α_{fa}	share of value-added for factor f in activity a
β_{ch}	share in household h consumption spending of commodity c
θ_{ac}	yield of output c per unit of activity a

Variables

P_c	market price of commodity c
PA_a	price of activity a
Q_c	output level in commodity c
QA_a	level of activity a
QF_{fa}	demand for factor f from activity a
QH_{ch}	consumption of commodity c by household h
YF_{hf}	income of household h from factor f
WF_f	price of factor f
YH_h	income of household h

---

[4]For a discussion of style in economic modeling, see Kendrick (1984).

Equations

Production and Commodity Block

Activity Production Function

$$QA_a = ad_a \cdot \prod_{f \in F} QF_{fa}^{\alpha_{fa}} \qquad a \in A \tag{1}$$

Factor Demand

$$WF_f = \frac{\alpha_{fa} \cdot PA_a \cdot QA_a}{QF_{fa}} \qquad f \in F, a \in A \tag{2}$$

Activity Price

$$PA_a = \sum_{c \in C} \theta_{ac} \cdot P_c \qquad a \in A \tag{3}$$

Commodity Output

$$Q_c = \sum_{a \in A} \theta_{ac} \cdot QA_a \qquad c \in C \tag{4}$$

Institution Block

Factor Income

$$YF_{hf} = shry_{hf} \cdot WF_f \cdot \sum_{a \in A} QF_{fa} \qquad h \in H, f \in F \tag{5}$$

Household Income

$$YH_h = \sum_{f \in F} YF_{hf} \qquad h \in H \tag{6}$$

Household Demand

$$QH_{ch} = \frac{\beta_{ch} \cdot YH_h}{P_c} \qquad c \in C, h \in H \tag{7}$$

System Constraint Block

Factor Market Equilibrium

$$\sum_{a \in A} QF_{fa} = qfs_f \qquad f \in F \tag{8}$$

Output Market Equilibrium

$$Q_c = \sum_{h \in H} QH_{ch} \qquad c \in C \tag{9}$$

Price Normalization Equation

$$\sum_{c \in C} cwts_c \cdot P_c = cpi \tag{10}$$

DATA BASE

The data base of the model is presented in Table 2.

Table 2—Social accounting matrix for Exercise 1

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	TOTAL
AGR-A			125						125
NAGR-A				150					150
AGR-C							50	75	125
NAGR-C							100	50	150
LAB	62	55							117
CAP	63	95							158
U-HHD					60	90			150
R-HHD					57	68			125
TOTAL	125	150	125	150	117	158	150	125	

TASK

With the help of an ASCII editor, input the above model using GAMS syntax. Solve the model in GAMS and verify that the solution can replicate the above SAM (Table 2).

HINTS AND SUGGESTIONS

1. Before attempting to do this exercise, the reader should be familiar with GAMS, at least at the level of the tutorial Chapter 2 in the user's guide (Brooke, Kendrick, Meeraus, and Raman 1998, 5–28). The rest of the guide is also an indispensable reference.
2. You may run the input file in GAMS at any point in the process of constructing the model. If the model is incomplete and hence not solved, GAMS will nevertheless check that the input conforms with its syntax, report any errors, and, in the absence of errors, carry out other instructions, including displays. To catch errors at an early stage, it is often helpful to inspect the results of displays of elements (sets, variables, and parameters), that have been defined via operations.
3. A timesaving device when developing the model is to use mechanical searches for text segments, including "****," which is used to indicate errors in the output file of GAMS. (By default the output file is called myfile.lst if the input file is called myfile or myfile.gms.)
4. The above mathematical statement is provided in a format that can be easily implemented in GAMS. It is advisable to use the same notation (subject to various minor transformations) since this will save effort (no additional notation is needed) and make it easier to move between the mathematics and GAMS.[5]

[5] Note that GAMS is case-insensitive. Nevertheless, it is easier to read a GAMS statement where the distinction between variables and parameters is evident, for example with parameters in lower case and variables in upper case (the convention followed in this manual).

For example, the first equation, with PRODFN as its declared name, may be input as follows:

```
PRODFN(A)..
                QA(A) =E= ad(A)*PROD(F, QF(F,A)**alpha(F, A));
```

5. In order to facilitate SAM-related computations, it is helpful to generate a global set, here named AC, including all elements in the sets for factors, activities, commodities, and households. Sets for the latter items are subsequently declared and defined as subsets of the global set.
6. When building CGE models, it is often useful to have identical sets with different names. In GAMS, the ALIAS command may be used to create a set that is identical to a set that already has been defined. (In the suggested answer, ALIAS is used to define sets identical to AC, C, and F.)
7. The models in this volume are formulated as a set of simultaneous equations and solved using a solver for nonlinear mixed-complementarity problems (PATH or MILES).[6] Alternatively, simultaneous-equation models may be solved as nonlinear optimization problems. To follow this approach you may define an additional equation, OBJFN as:

   ```
   OBJFN..      OBJ =E= DUMMY**2;
   ```

   where OBJ is an unconstrained variable, and DUMMY is a non-negative variable. The solution statement should be changed to SOLVE CGE1 USING NLP MINIMIZING OBJ. In this setting, the other equations would define the constraint set with only one feasible solution (identical to the solution to the corresponding simultaneous-equation model). OBJ is minimized when DUMMY has a value of zero.
8. Do not input the row and column totals of the SAM. Instead, to remove one source of errors and model infeasibility, compute row and column totals and check that they are identical.
9. After entering the SAM, start using it to define the parameter values and initial variable levels. Initial variable levels are helpful for two reasons. First, they give the GAMS solver a good starting point, facilitating its search for the solution. Second, they make it easier to pinpoint the reasons why a model fails to generate the benchmark equilibrium. The latter issue is discussed further under point 12.
10. Note that parameters (in this model, ad_a, for example) may be defined using preceding definitions. If this approach is fol-

[6] In the general case, a mixed-complementarity problem consists of a set of simultaneous equations that are a mixture of strict equalities and inequalitites, with the latter linked to bounded variables. The current model is a special case since all equations are strict equalities. For details and a mathematical definition, see Rutherford (1995).

lowed, it matters in which order the parameters and initial variable values are defined.

11. Assume that base-year factor and output prices are at unity and assign parameter and variable quantities on this basis. (This amounts to choosing the unit for each real flow so as to assure that its corresponding price is one.)
12. If you have implemented the model correctly, there should be no "significant" discrepancies between left-hand and right-hand sides of the equations when GAMS plugs in your parameter values and initial variable levels. You check this by looking for three asterisks (***) in the "equation listing." If the left-hand side value (indicated as "LHS = <value>") is "significantly" different (let's say by more than 1.0E-5) from the right-hand side value in the preceding equation (after =E=), a problem exists with the definitions of the parameters and variables that appear in the equation in question. Errors may be caught by displaying the values of all parameters and variables in any problematic equation and checking whether the values are compatible with the SAM.
13. The number of (single) equations and (single) variables reported as part of the "model statistics" should be identical (at 24 for the proposed solution).[7]
14. A value of unity for all factor and commodity prices (that were initialized at this level) is a reliable indicator that the initial model solution replicates the initial equilibrium as captured by the initial SAM. To test that the model is robust, it is a good idea to solve it a couple of times with different values for selected parameters. To check that the model indeed is homogeneous, the initial value of *cpi* may be multiplied by some positive factor. Compared to the base, the prices of the model solution should all be changed by the value of the factor, while all quantities should stay unchanged.
15. In addition to model statement and base solution, the GAMS input file with the suggested answer includes a LOOP where two simulations are carried out: the BASE and a simulation for which the capital stock is increased by 10 percent. A set of report parameters are created. For each model solution they show (1) values for the factor supply parameter (which was changed in the experiment); (2) solution values for all model variables; (3) a SAM that is defined using data from each model solution; and (4) percentage changes from the base solution for

---

[7] By default, the GAMS variable count also includes variables that are fixed (have exogenous values) unless the model attribute *holdfixed* has been specified with a value of one, in which case only endogenous variables are included in the count (see Brooke et al. 1998, 74–76). If the model includes fixed variables, this attribute makes it straightforward to verify that the number of endogenous variables and equations is equal. In this manual, fixed variables and the *holdfixed* attribute appear in the proposed GAMS solutions to Exercises 3–5.

model variables and SAM cells. The SAM parameters provide data on the budgets of all agents and markets in the model in a concise format. For a more comprehensive set of report parameters, see the GAMS listing of the Cameroon CGE model in Chapter 3 of Devarajan, Lewis, and Robinson (1994).

16. This first exercise may be the toughest one; it is certainly the one requiring the largest amount of new GAMS code. It is important that you try to do it *before* checking the suggested answer, because "learning by doing" is the name of the game. However, keep in mind that although the suggested answer tries to embody good modeling practices, it is merely a *suggested* answer. Different formulations may seem preferable to other users. In many ways, "style" in modeling and language includes room for taste differences.

APPENDIX: EXERCISE A1

The notation of the model of Exercise 1 is set-driven, that is, reference is frequently made to sets and set indexes.[8] The purpose of this exercise is to provide practice in interpreting what is hidden behind the veil of set notation and to demonstrate the gains from set notation in the form of more concise and more easily modified models. For example, the disaggregation of models based on set notation can be changed simply by varying the set definitions, the SAM, and other data, without any other changes in the input file.[9] The starting point for this Exercise is the GAMS input file with your correct answer to Exercise 1.

TASK In the GAMS statement, rewrite two of the equations in Exercise 1, PRODFN and FACDEM, in "longhand," that is, instead of making reference to sets and indexes, refer to specific set elements by name. Leave the rest of the model statement unchanged. Run the model and check that the solution is identical to Exercise 1.

HINTS

1. You have to declare and define a total of six equations. (In the suggested answer, they are named PRODFN1, PRODFN2, FACDEM1, FACDEM2, FACDEM3, and FACDEM4.) Put asterisks in the first character position of the lines where reference is made to the equations PRODFN and FACDEM (their definitions and declarations).[10]
2. Write the production function for the agricultural activity as follows:

```
PRODFN1..   QA('AGR-A') =E=
      ad('AGR-A')*QF('LAB','AGR-A')**alpha('LAB','AGR-A')
        *QF('CAP','AGR-A')**alpha('CAP','AGR-A').
```

---

[8] Recall that subscripts (but not superscripts) are set indexes.

[9] If one more production activity and commodity is added, the complete current model in longhand would require six new equations and modifications for five old equations. To add one more activity and commodity in the set-driven statement would merely require the addition of a couple of additional lines in the set definitions. Irrespective of approach, it would be necessary to further disaggregate the SAM.

[10] When an asterisk (*) is put in the first character position, GAMS ignores the rest of the line but reproduces it in the output file. This is a useful device for including short comments or excluding unused parts of a program without deleting them. (An alternative approach, preferable for longer comments, is to block off a section with $ONTEXT and $OFFTEXT before and after the section, respectively. The two "dollar" statements must start in the first character positions of their respectively lines.)

EXERCISE 2

In this exercise, intermediate demands are added to the CGE model presented in Exercise 1.

DATA BASE The new SAM is shown in Table 3. New payment flows, representing payments for intermediate goods, have been added in the cells at commodity row and activity column intersections. The accounts in the SAM are unchanged.

TASKS

1. Mathematical statement: Modify the mathematical statement so that the model incorporates intermediate demands. For both sectors, assume Leontief technology, that is, that a fixed input quantity is needed per unit of output.
2. GAMS: After having assured yourself that the answer to Task 1 is without errors (compare it to the suggested answer), implement the model in GAMS. This involves all or parts of the following: modifying the SAM; adding/modifying declarations and definitions for sets, parameters, variables (for this the "definition" involves defining the initial levels), and equations; displaying (and checking) the results of new computations; solving the model without errors for the base case and for a simple experiment (the latter to check that the model is robust); and confirming that it replicates the base data. Make sure that for each new element (set, variable, parameter, or equation), you go through the same steps as for the corresponding elements already present in the initial model.

Table 3—Social accounting matrix for Exercise 2

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	TOTAL
AGR-A			225						225
NAGR-A				250					250
AGR-C	60	40					50	75	225
NAGR-C	40	60					100	50	250
LAB	62	55							117
CAP	63	95							158
U-HHD					60	90			150
R-HHD					57	68			125
TOTAL	225	250	225	250	117	158	150	125	

Once the model is calibrated to the SAM, run the same experiment you did for Exercise 1.

HINTS AND SUGGESTIONS

1. Mathematical Statement

a. When modifying the statement, check that the changes in the number of variables and equations are equal (so that the number of variables and independent equations remains equal). Compared to the Exercise 1 model, the suggested model in Exercise 2 has six additional equations and variables, the total number being 30.

b. The suggested answer includes the following new elements:

Parameters

ica_{ca}	quantity of c as intermediate input per unit of activity a

Variables

PVA_a	value-added (or net) price of activity a
$QINT_{ca}$	quantity of commodity c as intermediate input in activity a

There are no new sets but two new equations for value-added prices and intermediate demands. Some changes in other equations are also needed.

2. GAMS

a. Be systematic when you modify the model: for every parameter/variable you declare, make sure you don't forget to include it in the equations or to define and display its value/initial level. Check that the displayed values coincide with the values you expect.

b. Note that both PVA_a and P_c cannot have initial (or equilibrium) levels of unity. The suggested answer follows the convention of keeping the initial value of P_c at unity and defining PVA_a at a level corresponding to the SAM payment to primary factors per unit of the activity.

c. In the same way as in Exercise 1, define the parameter ad_a so that the production (or, more precisely, value-added) function for activity a on its own will generate the base level for QA_a.

d. Pay special attention to Hint 12 in Exercise 1 for ways to track down the reasons why the model fails to replicate the base-year equilibrium.

EXERCISE 3

Few applied CGE models fall short of explicitly covering savings and investment. In our gradual process of constructing an applied model, we start by adding this aspect, using the CGE model of Exercise 2 as our starting point. Moreover, in the previous models, the wage (price) of each factor was assumed to be uniform across all activities that used that factor. In other words, every activity paid the average wage. In the real world, wages tend to be "distorted" in the broad sense that they differ across activities. A treatment that permits this variation (with no distortions as a special case) is also introduced in this exercise. We will assume that wages are distorted for labor but uniform across activities for capital, in a setting with full (or fixed) employment for both factors.

When doing the exercise, follow one general aspect of good modeling practice: introduce changes in different areas one at a time.

DATA BASE

The SAM, displayed in Table 4, includes one new account called savings-investment (S-I).[11] Its row receives payments from the household (the only saver in this simple economy); its column shows spending on commodities used for investment. Note that investment is defined in terms of the commodities used in the production of the capital stock, not the activity of destination (the activity that receives the investment goods as an addition to its capital stock). This means that the model only applies to a period so short that there is not enough time for new investments to provide additional production capacity. For a model relevant to a longer time period (for example a multiperiod model), it would also be necessary to consider explicitly the resulting changes in capital stock.

For labor, the number of workers employed is 100 for agriculture and 50 for nonagriculture. For capital, quantities are assigned on the assumption that the wage is unity for both activities. There are no changes in the SAM associated with the change in the factor treatment.

TASKS

1. Mathematical Statement

Expand the mathematical statement so that savings-investment is included and each activity pays fixed shares of average base wages for labor and capital. Proceed in two steps, starting with the savings-investment aspect. Introduce the changes in the factor treatment when you have assured yourself that the first step was accomplished without error.

[11] The change in the assumption about the capital market is not linked to any change in the SAM since the SAM merely reports payment flows. That is, the SAM does not say anything about the behavioral rules of the economy, including the workings of the capital market.

Table 4—Social accounting matrix for Exercise 3

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD	S-I	TOTAL
AGR-A			250							250
NAGR-A				305						305
AGR-C	60	40					50	75	25	250
NAGR-C	40	60					100	50	55	305
LAB	72	80								152
CAP	78	125								203
U-HHD					80	120				200
R-HHD					72	83				155
S-I							50	30		80
TOTAL	250	305	250	305	152	203	200	155	80	

For savings-investment, assume the following: (a) household income is allocated in fixed shares to savings and consumption; (b) investment is savings-driven, that is, the value of total investment spending is determined by the value of savings; and (c) investment spending is allocated to the two commodities in a manner such that the ratio between the quantities is fixed. Together, assumptions (b) and (c) mean that when savings values and/or the prices of investment commodities change, there is a proportional adjustment in the quantities of investment demand for each commodity, generating an investment value equal to the savings value.

For the factor markets (both labor and capital), assume that each activity pays an endogenous wage expressed as the product of an endogenous (economywide) wage variable (for the base equal to the average wage) and an exogenous distortion factor. For the special case of no distortions, the distortion factor is equal to one for all activities. In each factor market, variations in the average wage clear the market.

The set of equilibrium conditions that is functionally dependent now includes not only the commodity market equilibrium conditions, but also the savings-investment balance. It would be possible to drop one of these equations. In the suggested answer, another approach is selected. Instead of dropping one of these equations, a variable called WALRAS is introduced in the savings-investment balance. This approach is commonly used for this class of models. The model still has an equal number of variables and equations. If the model works, the savings-investment balance should hold, that is, the value of WALRAS should be zero.

2. GAMS After having produced an error-free mathematical statement, implement the model in GAMS, using the same systematic approach that was developed for Exercise 2. Proceed in two steps, starting with the introduction of savings-investment. Once you have confirmed that the base solution is able to replicate the base data set (except for assumed labor employment levels), proceed with the changes for factors. When the base solution also works well with this change, solve for the experiment with a 10 percent increase in the capital stock. Verify that the value for WALRAS remains (very close to) zero for this solution as well.

HINTS AND SUGGESTIONS

1. Mathematical Statement

a. Introduce the changes in a stepwise manner, in each step keeping track of changes in the number of variables and equations.
b. For the savings-investment modification, the following changes may accomplish the task:
   (i) Parameters: Introduce new parameters for household savings shares (called $\overline{mps_h}$) and base-year sectoral investment quantities (called $\overline{qinv_c}$);
   (ii) Variables: Add new variables for quantities of investment demand and a factor introducing proportional changes in investment quantities (referred to as $QINV_c$ and $IADJ$, respectively).
   (iii) Equations: Include new equations to define $QINV_c$ and to impose balance between savings and investment values. The investment equation may be written as $QINV_c = \overline{qinv_c} \cdot IADJ$.
c. The change in the treatment of factor markets may involve the following:
   (i) Parameters: Define and declare a new distortion factor ($wfdist_{fa}$) that represents the ratio between the wage for factor f in activity a and the average wage for factor f.
   (ii) Equations: To assure that payments for factors are made at distorted wages, multiply the average wage variable (WF_f) by the distortion factor in every equation where the wage variable appears. (The definition of the distortion factor implies that $WF_f \cdot wfdist_{fa}$ indeed defines the wage for factor f in activity a.)

2. GAMS

Once again, proceed in two steps. Before introducing savings and investment, add S-I to elements in the set AC and use the new SAM. For factors quantities and wages, you may go through the following steps: (1) on the basis of the information in the data base and the stated assumptions, define initial levels for the activity-specific factor demand variable (QF_{fa}) and the factor supply parameter (qfs_f); (2) define initial levels for the average wage variable (WF_f) and activity-specific wages (an auxiliary parameter that only is used to facilitate calibration) in the proposed solution called wfa_{fa}; (3) define the wage distortion parameter ($wfdist_{fa}$) as the ratio between wfa_{fa} and WF_f; and (4) for each activity-factor combination, verify that the product $Wf_f \cdot wfdist_{fa} \cdot QF_{fa}$ equals the SAM payment from the activity to the factor.[12]

The suggested GAMS model has 34 equations and variables.

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[12] The market-clearing economywide wage variable was initialized at the level of the average base wage. Generally speaking, it will not coincide with the economywide average wage for any experiment unless (1) $wfdist_{fa}$ equals one for all activities and/or (2) there is no change in the employment shares for the different activities. (This is confirmed by the results for Exercise 3.)

EXERCISE 4

Up to now, the modeled economy has not included a government, an essential actor in applied policy analysis. This defect is remedied in this exercise. The government of the model earns its revenues from income and sales taxes and spends it on consumption and transfers to households. Government savings is the difference between its revenues and spending.

In Exercise 3, we introduced activity-specific wage distortions. For both factors, we assumed full employment, free mobility across activities, and a flexible market-clearing wage. In this exercise, we will instead assume the following: (1) for labor: unemployment with fixed, activity-specific real wages and the quantity of labor supply as the market-clearing variable; and (2) for capital: full employment but no mobility between activities and a flexible market-clearing wage for each factor-activity combination.

DATA BASE

The model is built around the SAM shown in Table 5. Labor employment quantities are the same as for Exercise 3 (100 for agriculture and 50 for nonagriculture). The introduction of the government is behind the changes in the SAM structure. (The changes for the factor markets require no changes in the SAM.) There are new accounts for the government (GOV) and the two tax types, taxes on incomes (YTAX) and sales (STAX). In the tax rows, income taxes are collected from the household and sales taxes from the commodity accounts (AGR-C and NAGR-C). In the tax columns, this income is passed on to the government. The government column shows that the government uses this revenue to cover the cost of government commodity consumption (payments to AGR-C and NAGR-C), transfers to the households (payments to U-HHD and R-HHD), and (negative) government savings (payment to S-I). Note that in the rows of the commodity accounts, the demanders buy commodities at market prices; in the columns of the commodity accounts, these payments are split between the sales tax account and the activities (paid for output valued at producer prices).

One important part of government consumption, government payment for the labor services of its administrators and other employees, does not appear explicitly in the SAM. These government employees may be viewed as working for a government service activity that produces a commodity that is purchased by the government (institution) account. The government-service activity-commodity pair in this SAM is part of the nonagricultural activity and its commodity. In more disaggregated, real-world SAMs, the government service activity and commodity typically have their own accounts.

Table 5—Social accounting matrix for Exercise 4

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP
AGR-A			255			
NAGR-A				350		
AGR-C	66	44				
NAGR-C	44	66				
LAB	72	105				
CAP	73	135				
U-HHD					95	125
R-HHD					82	83
GOV						
S-I						
YTAX						
STAX			25	33		
TOTAL	255	350	280	383	177	208

	U-HHD	R-HHD	GOV	S-I	YTAX	STAX	TOTAL
AGR-A							255
NAGR-A							350
AGR-C	55	77	11	27			280
NAGR-C	110	55	47	61			383
LAB							177
CAP							208
U-HHD			25				245
R-HHD			5				170
GOV					25	58	83
S-I	60	33	–5				88
YTAX	20	5					25
STAX							58
TOTAL	245	170	83	88	25	58	

TASKS

1. Mathematical Statement

Present a statement for a model based on the above SAM that includes a government and has the proposed treatment for factor markets. Implement the changes in the two areas step by step, starting with the government. For the factors, the detailed assumptions were stated in the introduction to this exercise. For the government, assume the following:

a. The income tax is a fixed share of the gross income of each household. A fixed share of post-tax income is saved and the rest is spent on consumption.

b. Sales taxes are fixed shares of (mark-ups on) producer commodity prices.
c. The government consumes fixed commodity quantities, paying market prices (including the sales tax). Government transfers to the households are CPI-indexed, that is, they can simply be fixed in nominal terms. (Indexation to the CPI is automatic since the CPI level is fixed via the price normalization equation.[13])
d. Government savings is a residual, assuring balance between government outlays (including savings) and revenues. It is computed as the difference between expenditures (excluding savings) and revenues.

2. GAMS Once you have produced a correct mathematical statement, implement the model in GAMS in two steps. Make sure that in each step the model can replicate the data base and solves for an experiment where the quantities of government consumption of each commodity are increased by 20 percent. Analyze the impact of this change.

HINTS AND SUGGESTIONS

1. Mathematical Statement

To model the government, go through the following steps:

a. Sets: A new set, I (with an identical set named I'), defines institutions (currently the two households and the government; the rest of the world will be added in Exercise 5). It is referred to in the modeling of transfers between institutions.
b. Parameters: The new parameters, with suggested notation parenthesized, define government commodity consumption (qg_c), sales and income tax rates (tq_c and ty_h, respectively), and transfers from institution i' to institution i ($tr_{ii'}$). The transfer parameter captures transfers from the government to the households. In the equations where the parameter appears, reference is made explicitly to the relevant subset (H) and elements (GOV).[14]
c. Variables: The new variables, with the symbols in parenthesis, denote producer prices exclusive of the sales tax (PX_c), government revenue (YG), and government expenditures (EG). The sales tax introduces a wedge between the price received by the producers (PX_c) and that paid by the demanders. The latter

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[13] If government transfers and/or the labor wage are fixed, the model is, strictly speaking, no longer homogeneous of degree zero in prices. If you would like to maintain homogeneity, multiply government transfer parameters and the labor wage (but not the capital wage) by *cpi.*

[14] Alternatively, it would have been possible to declare this as a government transfer parameter with only the receiving set of institutions, *h,* in its domain. However, in a more complex model with many paying institutions, this approach would be tedious, requiring the definition of a separate parameter for each paying institution. The advantage of defining it over broadly defined sets of paying and receiving institutions is increased flexibility—one single parameter can handle a wider variety of contexts and fewer changes are needed elsewhere in the model. This will be evident in Exercise 5, where the rest of the world is added to the set of paying institutions, transferring money to both households and the government.

price includes the sales tax (the old symbol P_c is used to define this price). Thus, one important task is to change the variable P_c in the Exercise 3 model to PX_c in the current model whenever reference is made to what the producer receives (and not to what the demander pays).

d. Equations: New equations are needed to define government revenue and expenditures. Modifications are introduced in the equations for household income (government transfers are a new income source), household consumption demand (owing to the presence of income taxes), commodity market equilibrium (to account for government consumption), and the savings-investment balance (since the government represents a new source of savings).

For factors, a relatively flexible approach is suggested. The changes are

a. The parameters $wfdist_{fa}$ and qfs_f are turned into variables, written as $WFDIST_{fa}$ and QFS_f, respectively.
b. Among the factor wage and quantity variables, the following are fixed: $WFDIST_{lab,a}$, WF_{lab}, $QF_{cap,a}$, and WF_{cap}.

This approach is relatively flexible because, by selectively fixing factor wage and quantity variables, it can handle a variety of closure rules (including the one used in Exercise 3).

2. GAMS To model the government, the following hints may facilitate your task:

a. Augment the set AC with accounts for the government and the two tax types. Declare and define the new set for institutions.
b. Let the initial values be unity for all prices except PVA_a and P_c (that is, PX_c is among the prices with an initial value of unity). For any activity a, PVA_a may be defined using the initial activity price, PA_a, and data in the SAM activity column.
c. Calibrate the rate of the sales tax (tq_c) as the ratio between the tax payment *and* output value excluding the sales tax.
d. Given the values of tq_c and PX_c, you can compute the initial value of P_c.
e. Given that the commodity market price (P_c) paid by the demanders is no longer at unity, it is now necessary to explicitly consider this price when computing values for parameters and variables linked to commodity quantities.
f. Note that the household savings rate (mps_h) should now be computed as the ratio between household savings and household *disposable* (post-tax) income.

For the factor markets, the changes in GAMS follow from the changes in the mathematical statement in a straightforward manner. The suggested model has 38 variables and equations. The suggested GAMS solution illustrates the use of scalars and IF statements to facilitate shifts between alternative closures for the savings-investment balance and factor markets (see Brooke et al. 1998, 152–154).

EXERCISE 5

In this final exercise, we complete the model by adding the rest of the world (RoW). Interaction with the RoW takes place in the form of imports, exports, and transfers. Crucially, for demanders, imports and domestic output sold domestically are assumed to be imperfect substitutes. Similarly, for producers, imperfect transformability is assumed between exports and domestic output sold domestically.[15] Compared to the alternative of perfect substitutability (which, for any given commodity, only permits one-way trade), this treatment tends to generate more realistic responses by domestic prices, production, and consumption to changes in international prices. The treatment of factor markets is the same as for Exercise 4.

In combination with an appropriately disaggregated SAM, and data for labor employment and elasticities, the model that is the output of this exercise may provide the starting point for real-world applied policy analysis. However, it is highly likely that changes are needed to better reflect the structure of the modeled economy. Such changes, may, for example, include the introduction of price controls and other features that invalidate the assumption that flexible prices clear perfectly competitive markets. In addition, available production and consumption elasticities would typically suggest that the Cobb-Douglas functions should be replaced by more flexible (and complex) functional forms.

The task of implementing the model from scratch on the basis of stated assumptions is quite complex. Hence, not only the SAM, but a complete mathematical statement with brief comments on new features, is provided.

DATA BASE

The data base for the model consists of the SAM found in Table 6, unchanged data for labor employment, and trade elasticities. The values used are 0.7 for the elasticity of substitution between imports and domestic sales of the nonagricultural commodity, and 2 for the elasticity of transformation between exports and domestic sales of the agricultural commodity.

The SAM itself includes two new accounts, for the rest of the world (ROW) and for import tariffs (TAR). The row of the ROW account shows that our spending on imported commodities is the only income source of the RoW in its dealings with our country; the column of the same account shows that the receipts of our country from ROW consist of

[15] Imperfect substitutability and transformability may arise from differences in physical quality, differences in time and place of availability, and from aggregation biases.

Table 6—Social accounting matrix for Exercise 5

	AGR-A	NAGR-A	AGR-C	NAGR-C	LAB	CAP	U-HHD	R-HHD
AGR-A			279					
NAGR-A				394				
AGR-C	84	55					30	49
NAGR-C	50	99					165	92
LAB	72	105						
CAP	73	135						
U-HHD					95	125		
R-HHD					82	83		
GOV								
S-I							70	40
YTAX							20	5
STAX			10	20				
TAR				39				
ROW				105				
TOTAL	279	394	289	558	177	208	285	186

	GOV	S-I	YTAX	STAX	TAR	ROW	TOTAL
AGR-A							279
NAGR-A							394
AGR-C	13	28				30	289
NAGR-C	67	85					558
LAB							177
CAP							208
U-HHD	25					40	285
R-HHD	5					16	186
GOV			25	30	39	15	109
S-I	–1					4	113
YTAX							25
STAX							30
TAR							39
ROW							105
TOTAL	109	113	25	30	39	105	

export revenues and transfers to the households and the government.[16] The payments from ROW to S-I is foreign savings or the current account deficit, that is, the difference between our country's current (noncapital) foreign exchange expenditures and earnings.

MATHEMATICAL STATEMENT

The bulk of this statement consists of the model equations (a total of 27), divided into "blocks" for prices, production and commodities, institutions, and system constraints. Explanatory boxes are provided below each equation. New equations and other changes from the model in Exercise 4 are explained. The statement starts with alphabetical lists of sets, parameters, and variables that should serve as a reference as the reader goes through the equations.

Notation

Sets

$a \in A$	activities
$c \in C$	commodities
$c \in CM\ (\subset C)$	imported commodities
$c \in CNM\ (\subset C)$	nonimported commodities
$c \in CE\ (\subset C)$	exported commodities
$c \in CNE\ (\subset C)$	nonexported commodities
$f \in F$	factors
$h \in H\ (\subset I)$	households
$i \in I$	institutions (households, government, and rest of world)

Parameters

ad_a	production function efficiency parameter
aq_c	shift parameter for composite supply (Armington) function
at_c	shift parameter for output transformation (CET) function[17]
cpi	consumer price index
$cwts_c$	commodity weight in CPI
ica_{ca}	quantity of c as intermediate input per unit of activity a
mps_h	share of disposable household income to savings
pwe_c	export price (foreign currency)
pwm_c	import price (foreign currency)
$\underline{qg_c}$	government commodity demand
$\overline{qinv}_c$	base-year investment demand
$shry_{hf}$	share of the income from factor f in household h
te_c	export tax rate
tm_c	import tariff rate
tq_c	sales tax rate

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[16] Neither of the two commodities is both exported and imported. The phenomenon of two-way trade ("cross-hauling") is nevertheless commonly observed in the real world at the level of commodity aggregation used in applied models. It can be handled by the proposed approach without any modifications in the model structure.

[17] The acronym CET stands for constant elasticity of transformation.

$tr_{ii'}$	transfer from institution i' to institution i
ty_h	rate of household income tax
α_{fa}	value-added share for factor f in activity a
β_{ch}	share of commodity c in the consumption of household h
δ_c^q	share parameter for composite supply (Armington) function
δ_c^t	share parameter for output transformation (CET) function
θ_{ac}	yield of commodity c per unit of activity a
ρ_c^q	exponent ($-1 < \rho_c^q < \infty$) for composite supply (Armington) function
ρ_c^t	exponent ($1 < \rho_c^t < \infty$) for output transformation (CET) function
σ_c^q	elasticity of substitution for composite supply (Armington) function
σ_c^t	elasticity of transformation for output transformation (CET) function

Variables

EG	government expenditure
EXR	foreign exchange rate (domestic currency per unit of foreign currency)
$FSAV$	foreign savings
$IADJ$	investment adjustment factor
PA_a	activity price
PD_c	domestic price of domestic output
PE_c	export price (domestic currency)
PM	import price (domestic currency)
PQ_c	composite commodity price
PVA_c	value-added price
PX_c	producer price
QA_a	activity level
QD_c	quantity of domestic output sold domestically
QE_c	quantity of exports
QF_{fa}	quantity demanded of factor f by activity a
QFS_f	supply of factor f
QH_{ch}	quantity of consumption of commodity c by household h
$QINT_c$	quantity of intermediate use of commodity c by activity a
$QINV_c$	quantity of investment demand
QM_c	quantity of imports
QQ_c	quantity supplied to domestic commodity demanders (composite supply)
QX_c	quantity of domestic output
$WALRAS$	dummy variable (zero at equilibrium)
WF_f	average wage (rental rate) of factor f
$WFDIST_{fa}$	wage distortion factor for factor f in activity a
YF_{hf}	transfer of income to household h from factor f
YG	government revenue
YH_h	household income

Equations

Price Block

The introduction of foreign trade with product differentiation drastically enriches the price system—out of the six equations in this block, four (Equations 1–4) are new.

Import Price

$$PM_c = (1 + tm_c) \cdot EXR \cdot pwm_c \qquad c \in CM$$

$$\begin{bmatrix} \textit{import} \\ \textit{price} \\ \textit{(dom. cur.)} \end{bmatrix} = \begin{bmatrix} \textit{tariff} \\ \textit{adjust-} \\ \textit{ment} \end{bmatrix} \cdot \begin{bmatrix} \textit{exchange rate} \\ \textit{(dom. cur. per} \\ \textit{unit of for. cur.)} \end{bmatrix} \cdot \begin{bmatrix} \textit{import} \\ \textit{price} \\ \textit{(for. cur.)} \end{bmatrix} \tag{1}$$

Export Price

$$PE_c = (1 - te_c) \cdot EXR \cdot pwe_c \qquad c \in CE$$

$$\begin{bmatrix} \textit{export} \\ \textit{price} \\ \textit{(dom. cur.)} \end{bmatrix} = \begin{bmatrix} \textit{tariff} \\ \textit{adjust-} \\ \textit{ment} \end{bmatrix} \cdot \begin{bmatrix} \textit{exchange rate} \\ \textit{(dom. cur. per} \\ \textit{unit of for. cur.)} \end{bmatrix} \cdot \begin{bmatrix} \textit{export} \\ \textit{price} \\ \textit{(for. cur.)} \end{bmatrix} \tag{2}$$

The exogeneity of foreign-currency import and export prices indicates that we are modeling a country that is small relative to the relevant world markets (the "small-country" assumption). Note that Equations 1 and 2 only apply to imported and exported commodities, respectively.

Absorption

$$PQ_c \cdot QQ_c = \left[PD_c \cdot QD_c + (PM_c \cdot QM_c)_{|c \in CM}\right] \cdot (1 + tq_c) \qquad c \in C$$

$$[\textit{absorption}] = \left(\begin{bmatrix} \textit{domestic sales price} \\ \textit{times} \\ \textit{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \textit{import price} \\ \textit{times} \\ \textit{import quantity} \end{bmatrix} \right) \cdot \begin{bmatrix} \textit{sales tax} \\ \textit{adjustment} \end{bmatrix} \tag{3}$$

For each commodity, absorption—total domestic spending on the commodity at domestic demander prices—is expressed as the sum of spending on domestic output and imports, including an upward adjustment for the sales tax. The fact that this condition holds follows from the linear homogeneity of the composite supply (Armington) function (Equation 11; the condition is referred to as Euler's theorem). The import part only applies to imported commodities. The composite price, PQ_c, is paid by domestic demanders (households, the government, producers, and investors); hence it replaces P_c in all relevant equations. The composite price, implicitly defined by this equation, could easily be derived by dividing through by QQ_c. (See discussion of Equations 11 and 12 for further details.)

Domestic Output Value

$$PX_c \cdot QX_c = PD_c \cdot QD_c + (PE_c \cdot QE_c)_{|c \in CE} \qquad c \in C$$

$$\begin{bmatrix} \textit{producer} \\ \textit{price} \\ \textit{times} \\ \textit{domestic} \\ \textit{output quantity} \end{bmatrix} = \begin{bmatrix} \textit{domestic} \\ \textit{sales price} \\ \textit{times} \\ \textit{domestic} \\ \textit{sales quantity} \end{bmatrix} + \begin{bmatrix} \textit{export} \\ \textit{price} \\ \textit{times} \\ \textit{export} \\ \textit{quantity} \end{bmatrix} \tag{4}$$

For each commodity, domestic output value at producer prices is stated as the sum of the value of domestic output sold domestically and the export value (in domestic currency). This equation reflects the fact that the CET (constant-elasticity-of-transformation) function (Equation 14) is linearly homogeneous. The export part only applies to exported commodities. The producer price, PX_c, can be derived by dividing through by QX_c. Note that, in this model, the domestic output quantity is referred to as QX_c (as opposed to Q_c in earlier models). See discussion of Equations 14 and 15 for further details.

Activity Price

$$PA_a = \sum_{c \in C} PX_c \cdot \theta_{ac} \qquad a \in A$$

$$\begin{bmatrix} \textit{activity} \\ \textit{price} \end{bmatrix} = \begin{bmatrix} \textit{producer prices} \\ \textit{times yields} \end{bmatrix} \tag{5}$$

Value-added Price

$$PVA_a = PA_a - \sum_{c \in C} PQ_c \cdot ica_{ca} \qquad a \in A$$

$$\begin{bmatrix} \textit{value-} \\ \textit{added} \\ \textit{price} \end{bmatrix} = \begin{bmatrix} \textit{activity} \\ \textit{price} \end{bmatrix} = \begin{bmatrix} \textit{input cost} \\ \textit{per activity} \\ \textit{unit} \end{bmatrix} \tag{6}$$

Note that in this equation, there is a change in notation for the price applying to intermediate inputs (the composite supply price).

Production and Commodity Block

In this block, Equations 7–10 are unchanged compared to Exercise 4 (except for a minor notation change in Equation 10). Equations 11–16 are new. They allocate domestic supply of composite commodities between imports and domestic output, and transform domestic output to exports and domestic sales. Simpler expressions apply to commodities that are not imported and/or not exported.

Activity Production Function

$$QA_a = ad_a \cdot \prod_{f \in F} QF_{fa}^{\alpha_{fa}} \qquad a \in A$$

$$\begin{bmatrix} activity \\ level \end{bmatrix} = f \begin{bmatrix} factor \\ inputs \end{bmatrix} \tag{7}$$

Factor Demand

$$WF_f \cdot WFDIST_{fa} = \frac{\alpha_{fa} \cdot PVA_a \cdot QA_a}{QF_{fa}} \qquad f \in F, a \in A$$

$$\begin{bmatrix} marginal\ cost \\ of\ factor\ f \\ in\ activity\ a \end{bmatrix} = \begin{bmatrix} marginal\ revenue \\ product\ of\ factor \\ f\ in\ activity\ a \end{bmatrix} \tag{8}$$

Intermediate demand

$$QINT_{ca} = ica_{ca} \cdot QA_a \qquad c \in C, a \in A$$

$$\begin{bmatrix} inter\text{-} \\ mediate \\ demand \end{bmatrix} = f \begin{bmatrix} activity \\ level \end{bmatrix} \tag{9}$$

Output Function

$$QX_c = \sum_{a \in A} \theta_{ac} \cdot QA_a \qquad c \in C$$

$$\begin{bmatrix} domestic \\ output \end{bmatrix} = f \begin{bmatrix} activity \\ level \end{bmatrix} \tag{10}$$

Composite Supply (Armington) Function

$$QQ_c = aq_c \cdot \left(\delta_c^q \cdot QM_c^{-\rho_c^q} + (1 - \delta_c^q) \cdot QD_c^{-\rho_c^q} \right)^{\frac{-1}{\rho_c^q}} \qquad c \in CM$$

$$\begin{bmatrix} composite \\ supply \end{bmatrix} = f \begin{bmatrix} import\ quantity,\ domestic \\ use\ of\ domestic\ output \end{bmatrix} \tag{11}$$

The composite commodities are used by all domestic demanders. Imperfect substitutability between imports and domestic output sold domestically is captured by a CES (constant elasticity of substitution) aggregation function in which the composite commodity that is supplied domestically is "produced" by domestic and imported commodities, and enters this function as "inputs." Economically, this means that demander preferences over imports and domestic output are expressed as a CES function. This function, with a domain that is limited to elements in *CM,* is often called an Armington function after the originator of the idea of using a CES function for this purpose. The restriction on the value of ρ_c^q $(-1 < \rho_c^q < \infty)$ assures that the corresponding isoquant is convex to the origin, in terms of production economics equivalent to a diminishing technical rate of substitution.

Import-Domestic Demand Ratio

$$\frac{QM_c}{QD_c} = \left(\frac{PD_c}{PM_c} \cdot \frac{\delta_c^q}{1 - \delta_c^q} \right)^{\frac{1}{1 + \rho_c^q}} \qquad c \in CM$$

$$\begin{bmatrix} \textit{import-} \\ \textit{domestic} \\ \textit{demand ratio} \end{bmatrix} = f \begin{bmatrix} \textit{domestic-} \\ \textit{import} \\ \textit{price ratio} \end{bmatrix} \tag{12}$$

Equation 12 defines the optimal mix between imports and domestic output. Its domain is also limited to imported commodities. Together, Equations 3, 11, and 12 constitute the first-order conditions for cost-minimization given the two prices and subject to the Armington function and a fixed quantity of the composite commodity.

Composite Supply for Nonimported Commodities

$$QQ_c = QD_c \qquad c \in CNM$$

$$\begin{bmatrix} \textit{composite} \\ \textit{supply} \end{bmatrix} = \begin{bmatrix} \textit{domestic use of} \\ \textit{domestic output} \end{bmatrix} \tag{13}$$

For commodities that are not imported, the Armington function is replaced by the above statement, which imposes equality between "composite" supply and domestic output used domestically.

Output Transformation (CET) Function

$$QX_c = at_c \cdot \left(\delta_c^t \cdot QE_c^{\rho_c^t} + (1 - \delta_c^t) \cdot QD_c^{\rho_c^t} \right)^{\frac{1}{\rho_c^t}} \qquad c \in CE$$

$$\begin{bmatrix} \textit{domestic} \\ \textit{output} \end{bmatrix} = f \begin{bmatrix} \textit{export quantity, domestic} \\ \textit{use of domestic output} \end{bmatrix} \tag{14}$$

Imperfect substitutability between imports and domestic output sold domestically is paralleled by imperfect transformability between domestic output for exports and domestic sales. The latter is captured by Equation 14. The CET function, which applies to exported commodities, is identical to a CES function except for negative elasticities of substitution. The isoquant corresponding to the output transformation function will be concave to the origin given the restriction imposed on the value of ρ^t $(-1 < \rho_c^t < \infty)$. In economic terms, the difference between the Armington and CET functions is that the arguments in the former are inputs, those in the latter are outputs.

Export-Domestic Supply Ratio

$$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PD_c} \cdot \frac{1 - \delta_c^t}{\delta_c^t} \right)^{\frac{1}{\rho_c^t - 1}} \qquad c \in CE$$

$$\left[\begin{array}{c} \textit{export-} \\ \textit{domestic} \\ \textit{supply ratio} \end{array} \right] = f \left[\begin{array}{c} \textit{export-} \\ \textit{domestic} \\ \textit{price ratio} \end{array} \right] \tag{15}$$

Equation 15 defines the optimal mix between exports and domestic sales. Equations 4, 14, and 15 constitute the first-order conditions for maximization of producer revenues given the two prices (export and domestic) and subject to the CET function and a fixed quantity of domestic output.

One important difference between the equations for import demand (12) and export supply (15) is that the quantity demanded of the imported commodity (QM_c) is inversely related to the import price, whereas the quantity supplied of the exported commodity (QE_c) is directly related to the export price.

Output Transformation for Nonexported Commodities

$$QX_c = QD_c \qquad c \in CNE$$

$$\left[\begin{array}{c} \textit{domestic} \\ \textit{output} \end{array} \right] = \left[\begin{array}{c} \textit{domestic sales of} \\ \textit{domestic output} \end{array} \right] \tag{16}$$

For commodities that are not exported, the CET function is replaced by a statement imposing equality between domestic output sold domestically and domestic output.

Institution Block

This block is not changed compared to Exercise 4, except for the appearance of items related to interactions with the rest of the world (trade and transfers) in the definitions of household revenue and the revenue and expenditure of the government.

Factor Income

$$YF_{hf} = shry_{hf} \cdot \sum_{a \in A} WF_f \cdot WFDIST_{fa} \cdot QF_{fa} \qquad h \in H, f \in F$$

$$\left[\begin{array}{c} \textit{household} \\ \textit{factor} \\ \textit{income} \end{array} \right] = \left[\begin{array}{c} \textit{income} \\ \textit{share to} \\ \textit{household } h \end{array} \right] \cdot \left[\begin{array}{c} \textit{factor} \\ \textit{income} \end{array} \right] \tag{17}$$

Household Income

$$YH_h = \sum_{f \in F} YF_{hf} + tr_{h,gov} + EXR \cdot tr_{h,row} \qquad h \in H$$

$$\begin{bmatrix} \textit{household} \\ \textit{income} \end{bmatrix} = \begin{bmatrix} \textit{factor} \\ \textit{incomes} \end{bmatrix} + \begin{bmatrix} \textit{transfers from} \\ \textit{government \&} \\ \textit{rest of world} \end{bmatrix} \tag{18}$$

Household Consumption Demand

$$QH_{ch} = \frac{\beta_{ch} \cdot (1 - mps_h) \cdot (1 - ty_h) \cdot YH_h}{PQ_c} \qquad c \in C, h \in H$$

$$\begin{bmatrix} \textit{household} \\ \textit{demand for} \\ \textit{commodity c} \end{bmatrix} = f \begin{bmatrix} \textit{household income,} \\ \textit{composite price} \end{bmatrix} \tag{19}$$

Investment Demand

$$QINV_c = \overline{qinv_c} \cdot IADJ \qquad c \in C$$

$$\begin{bmatrix} \textit{investment} \\ \textit{demand for} \\ \textit{commodity c} \end{bmatrix} = \begin{bmatrix} \textit{base-year investment} \\ \textit{times} \\ \textit{adjustment factor} \end{bmatrix} \tag{20}$$

Government Revenue

$$\begin{aligned} YG &= \sum_{h \in H} ty_h \cdot YH_h + EXR \cdot tr_{gov,row} + \sum_{c \in C} tq_c \cdot \left(PD_c \cdot QD_c + (PM_c \cdot QM_c)_{|c \in CM} \right) \\ &\quad + \sum_{c \in CM} tm_c \cdot EXR \cdot pwm_c \cdot QM_c + \sum_{c \in CE} te_c \cdot EXR \cdot pwe_c \cdot QE_c \end{aligned} \tag{21}$$

$$\begin{bmatrix} \textit{govern-} \\ \textit{ment} \\ \textit{revenue} \end{bmatrix} = \begin{bmatrix} \textit{direct} \\ \textit{taxes} \end{bmatrix} + \begin{bmatrix} \textit{transfers} \\ \textit{from} \\ \textit{RoW} \end{bmatrix} + \begin{bmatrix} \textit{sales} \\ \textit{tax} \end{bmatrix} + \begin{bmatrix} \textit{import} \\ \textit{tariffs} \end{bmatrix} + \begin{bmatrix} \textit{export} \\ \textit{taxes} \end{bmatrix}$$

Government Expenditures

$$EG = \sum_{h \in H} tr_{h,gov} + \sum_{c \in C} PQ_c \cdot qg_c$$

$$\begin{bmatrix} \textit{government} \\ \textit{spending} \end{bmatrix} = \begin{bmatrix} \textit{household} \\ \textit{transfers} \end{bmatrix} = \begin{bmatrix} \textit{government} \\ \textit{consumption} \end{bmatrix} \tag{22}$$

System Constraint Block

This block defines the constraints that are satisfied by the economy as a whole without being considered by its individual agents. The model's *micro* constraints apply to individual markets for factors and commodities. With the few exceptions discussed below (for labor, exports, and imports), it is assumed that flexible prices clear the markets for all commodities and factors. The *macro* constraints apply to the government, the savings-investment balance, and the rest of the world. For the government, savings clear the balance, whereas the investment value adjusts to changes in the value of total savings. For the rest of the world, the alternatives of a flexible exchange rate or flexible foreign savings are permitted in the current formulation.

In this block, the rest-of-world constraint (Equation 25) is new while the commodity market and savings-investment balance (Equations 24 and 26) have been modified (compared to Exercise 4). The treatment of factor markets (Equation 23) is unchanged.

Factor Markets

$$\sum_{a \in A} QF_{fa} = QFS_f \qquad f \in F$$

$$\begin{bmatrix} \textit{demand for} \\ \textit{factor } f \end{bmatrix} = \begin{bmatrix} \textit{supply of} \\ \textit{factor } f \end{bmatrix} \tag{23}$$

For the two factors, the closure rules are the same as for Exercise 4: unemployment with fixed, activity-specific real wages for labor and fixed capital use for each activity. This is achieved by fixing the following variables at base values: $WFDIST_{lab,a}$, WF_{lab}, $QF_{cap,a}$, and WF_{cap}.

Composite Commodity Markets

$$QQ_c = \sum_{a \in A} QINT_{ca} + \sum_{h \in H} QH_{ch} + qg_c + QINV_c \qquad c \in C$$

$$\begin{bmatrix} \textit{composite} \\ \textit{supply} \end{bmatrix} = \begin{bmatrix} \textit{composite demand;} \\ \textit{sum of intermediate,} \\ \textit{household, government,} \\ \textit{\& investment demand} \end{bmatrix} \tag{24}$$

In the absence of foreign trade, the commodity market equilibrium condition in Exercises 1–4 equated output and domestic demand. This new equilibrium condition imposes equality in the composite commodity market with the demand side represented by all types of domestic commodity use while the supply comes from the Armington function (or its substitute for nonimported commodities) that aggregates imports and domestic output sold domestically. The variable PQ_c clears this market.

In addition to the composite commodity, the model includes quantity (and associated price) variables for the following commodities and activities: *QM, QE, QX, QD, QA*. These variables represent both the quantities supplied and demanded (that is, the equilibrium quantity has been substituted for the quantities supplied and demanded throughout the model). For exports and imports, the quantities demanded and supplied clear the markets (infinitely elastic world market demands and supplies at fixed foreign-currency prices). For the remaining three quantities, the associated price variables (*PX, PD,* and *PA*) serve the market-clearing role. (Exercise: Rewrite the model with separate supply and demand variables replacing *QM, QE, QX, QD, QA,* and a full set of equilibrium conditions for the corresponding markets.)

Current Account Balance for RoW (in Foreign Currency)

$$\sum_{c \in C} pwe_c \cdot QE_c + \sum_{i \in I} tr_{i,row} + FSAV = \sum_{c \in CM} pwm_c \cdot QM_c \tag{25}$$

$$\begin{bmatrix} export \\ revenue \end{bmatrix} = \begin{bmatrix} transfers \\ from\ RoW \\ to\ households \\ \&\ government \end{bmatrix} = \begin{bmatrix} foreign \\ savings \end{bmatrix} = \begin{bmatrix} import \\ spending \end{bmatrix}$$

The current-account equation (which is expressed in foreign currency) imposes equality between the country's earning and spending of foreign exchange. Foreign savings is equal to the current-account deficit. Careful counting of equations and variables in the current model would indicate that the number of variables exceeds the number of equations by one. This is related to the fact that the model includes two variables that may serve the role of clearing the current-account balance—the foreign exchange rate (*EXR*) and foreign savings (*FSAV*). The experiment for this Exercise (see below), assumes that *FSAV* is fixed.

Savings-Investment Balance

$$\sum_{h \in H} mps_h \cdot (1 - ty_h) \cdot YH_h + (YG - EG) + EXR \cdot FSAV$$
$$= \sum_{c \in C} PQ_c \cdot QINV_c + WALRAS \tag{26}$$

$$\begin{bmatrix} household \\ savings \end{bmatrix} + \begin{bmatrix} government \\ savings \end{bmatrix} + \begin{bmatrix} foreign \\ savings \end{bmatrix} = \begin{bmatrix} invest- \\ ment \\ spending \end{bmatrix} + \begin{bmatrix} WALRAS \\ dummy \\ variable \end{bmatrix}$$

Foreign savings, converted into domestic currency, appears as a new item in Equation 26. As long as either the exchange rate or foreign savings is fixed, their presence does not influence the savings-investment closure of the model, according to which the savings value determines the investment value.

Price Normalization

$$\sum_{c \in C} PQ_c \cdot cwts_c = cpi$$

$$\begin{bmatrix} price\ times \\ weights \end{bmatrix} = [CPI] \tag{27}$$

TASK Implement the model presented in the mathematical statement in GAMS, that is, calibrate it to the base, solve it to confirm that it calibrates, and implement a simple experiment where the world price (in foreign currency) of the agricultural commodity increases by 25 percent. Assume that the exchange rate is flexible. Analyze the impact.

As your starting point, use the file CGE4.GMS (the suggested answer to Exercise 4) and relevant parts of CGE5HLP.TXT. The latter includes the new SAM, as well as declarations and definitions for new parameters, variables, and equations. However, the numerous changes in definitions for old equations, parameters, and variables are not included.

HINTS It is suggested that you go through the following steps:

1. Carefully review the file CGE4.GMS, the mathematical statement for the Exercise 5 model, and the relevant part of CGEHLP.TXT
2. Make a copy of the file CGE4.GMS named CGE5.GMS
3. Carefully work through the file CGE5.GMS starting from the very beginning, introducing modifications when implied by the mathematical statement and the new SAM, and copying segments from CGE5HLP.TXT. In particular, copying and pasting the relatively complex definitions of parameters related to Armington and CET functions should be helpful.
4. When encountering problems, draw on suggestions in earlier exercises regarding how to debug the model. Rely on the GAMS user's guide as a reference.

Regarding initial values for price variables, it may be useful to note the following: In general, try to initialize as many prices as possible at unity. In the current model, this is the case for the capital wage and for the commodity prices PE_c, PM_c, PD_c, and PX_c. However, the possible presence of taxes and subsidies may impose divergence from this initialization rule for three other commodity prices: $tq_c > 0 \Rightarrow PQ_c > 1$; $tm_c > 0 \Rightarrow pwm_c < 1$; and $te_c > 0 \Rightarrow pwe_c > 1$. (However, according to the current SAM, $te_c = 0$ for all commodities.)

The suggested GAMS model has 49 variables and equations. The GAMS solution uses an additional scalar and two IF statements to select closure for the foreign exchange market. Moreover, a new report parameter is used to compute GDP at market prices in two alternative ways (as total final demand for domestic output, value at market prices; and as GDP at factor cost plus net indirect taxes).

GOOD LUCK!

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