

Student's Full Name: _____

Directions: Please read this page in all its entirety.

- Make sure your full name is written clearly at the top of this page
- Keep your work organized and legible
- Highlight or Box your final solutions
- **SHOW ALL YOUR STEPS! NO WORK = NO CREDIT!!**
- **Initial the integrity statement at the end of this Final Exam.**

You May...

- Use any of your lecture notes/videos (open-book, open-notes)
- Ask me any clarification questions (please don't assume anything!)

You May NOT...

- Receive or give help to any other students
- Post the questions online, discord, text-messaging, Instagram, or ... you get the idea.

Lastly, **good luck, I know you can do it!** 😊

Again, show all work that leads to your answers. Be sure your steps are in a logical sequence and that it is clear where each new step begins and ends. Highlight/box your solution.

1. (15 pts) Michael is very fascinated with how otters play in the water. Michael is watching an Otter named Quad, simply because one day Quad was on this huge rock that was 10 *ft* above Quad's favorite swimming hole, and dove down into the swimming hole and came back up all in a parabolic manner, with a fish in her mouth. Quad's distance from the surface of the water (in *ft*) after s seconds from her dive can be modeled by the following function:

$$Q(s) = \frac{10}{3}s^2 - 20s + 10$$

a. Find $Q(1)$ and interpret it's meaning in context of the story using a full sentence. (no calculus here, but **Show ALL Work.**)

b. It turns out that a fish was traveling tangent to Quad's $Q(1)$ position. Find the tangent line that the fish was following. (**Show ALL Work**)

c. What was Quad's maximum depth during this event? (use Calculus, **Show ALL Work**)

d. Sketch a picture of the scenario (Don't draw a graph! I want to see some creativity here.)

e. Turns out that Michael is not great at naming otters, what would you have named the otter?

2. (10 points) True/False with Justification! Please decide whether the following statements are true or false and **justify your decision** with mathematical reasoning.

a. Every function has both an absolute maximum and absolute minimum on any closed interval included in its domain.

b. The only critical values a function has are all the x -values such that $f'(x) = 0$.

c. If a given function is continuous and differentiable on an interval, The Mean Value Theorem implies that there is at least one point on the interval where the tangent line has the same slope as the average rate of change connecting the endpoints of the interval.

d. If c is a critical number of f and $f''(c) > 0$, then f has a local maximum value at $x = c$.

e. One example of a Strong Form is 0^0 .

3. (10 points) Find dy/dx for the following: (**Show ALL Work**)

a. $\cos(y) + x^3 + xy^2 = 42$

b. $\ln(y) - 5^{x-2} + 3y = -2$

4. (5 pts) My ZigZag sewing pattern. The other day I was sewing two pieces of fabric together using a zigzag sewing pattern. It turns out the needle's position (in millimeters *mm*) on the fabric after t milliseconds (*ms*) had passed can be modeled by the following function

$$N(t) = \sin(8t), 0 \leq t$$

- a. Find the velocity function $v(t)$. (**Show ALL Work**)

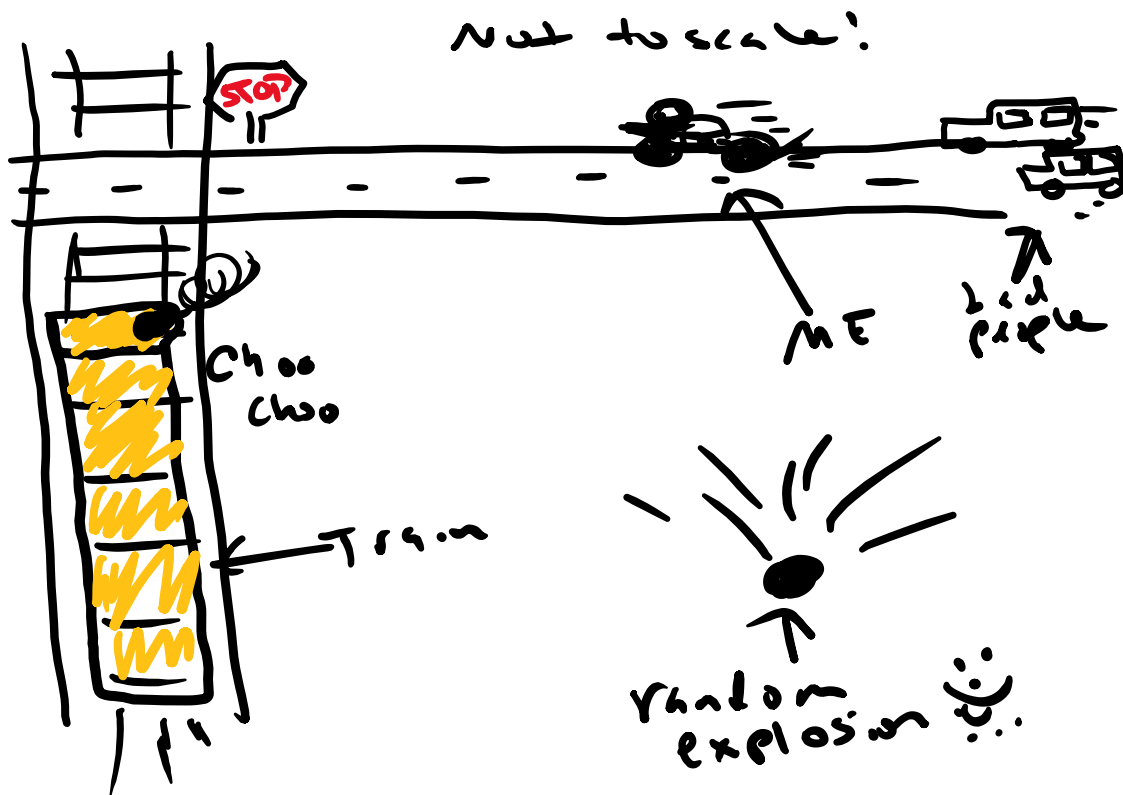
- b. Find $v(1)$ (**Show ALL Work**) and interpret it's meaning in a full sentence.

- c. Find the acceleration function $a(t)$. (**Show ALL Work**)

- d. Find $a(1)$ (**Show ALL Work**) and interpret it's meaning in a full sentence.

5. (15 points) Michael watches movies all the time, and if you watch movies too, you have probably watched some scene where a person in a vehicle is being chased and they just barely make it to an intersection where a train cuts off the people that are chasing.

Suppose Michael is one of these elite operatives and is traveling by motorcycle (way cooler!) and wants to use the train to cut off the people that are chasing him. Michael is traveling at 140 mph (safely?) and is currently 2 miles from the intersection. The freight train is traveling at 60 mph and is just slightly over $\frac{6}{7}$ (but we will say $\frac{6}{7}$) of a mile from the intersection. Most realistic illustration of the scenario below:



a. How fast is the distance between me and the train changing when I am 1 mile from the intersection (**Show ALL Work**, this one is tougher.)

b. Try (this might be tough too!) to sketch a better picture of the scenario.

6. (5 points) Evaluate the following limits by converting to strong form, then using L'Hopital's Theorem. (**Show ALL Work**, and do not use a table or graph.)

a. $\lim_{x \rightarrow 0^+} (x^3 \ln(3x))$

b. $\lim_{x \rightarrow 7} \left(\frac{x^2 - 49}{x^2 - 9x + 14} \right)$

7. (10 points) Let

$$g(t) = -\frac{2}{3}t^3 + \frac{42}{2}t^2 - 160t + 5$$

a. Find all the critical points of $g(t)$. (**Show ALL Work**)

b. Determine if any of the critical points, from part a, are local extremums and if so, determine whether it a local max or local min. By using the first or second derivative test. (**Show ALL Work**)

c. Find all the inflection points of $g(t)$. (**Show ALL Work**)

8. (15 points) Let

$$f(x) = 2x^2 - x + 3$$

- a. Using the definition of the derivative find the derivative of $f(x)$. (**Show ALL Work**)
- b. Using calculus, where on the function (as an ordered pair (x, y)) does $f(x)$ have the slope of the tangent line equal to zero? (**Show ALL Work**)
- c. Where on the domain of $[-1, 1]$ does the slope of the tangent line match the average rate of change on that interval? (**Show ALL Work**)

9. (10 points)

I recently bought some fencing to keep my pigs from venturing the neighborhood. I bought 100 meters of wire fencing, to make some rectangular area for them. I want to maximize their area by using an already pre-established back fence as one of the lengths.

- a. Sketch a picture of the scenario (better see pigs in it!)
- b. What dimensions of this pig oasis that I want to build that will maximize their area? (**Show ALL Work**)

10. (5 points) Use the quotient rule to find the derivative of the following function:

$$w(x) = \frac{5 \cos(3x)}{3 \sin(5x)}$$

Integrity Statement

READ CAREFULLY!

By initialing below, I am asserting that I did not cheat/plagiarize, help someone cheat/plagiarize, or post any of the Final Exam's questions onto any social media services. (Note: If you do not initial below, I will assume you cheated in some manner and give you a zero on the Final).

Initials: _____

Anyone who violates this integrity statement (meaning you cheated), will be given a zero for the exam and reported to Academic Student Affairs for Academic Dishonesty. Anyone who plagiarizes (taking credit for someone else's work) will be given a letter grade F for this class and reported to Academic Student Affairs for Academic Dishonesty.

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This concludes the Final Exam. Thank you again.