

2. The water is pumped into a spherical tank. The water starts to overflow when the content reaches 80% of its total volume. The total volume,  $V(H, R)$  of the spherical tank is given as

$$V(H, R) = \frac{\pi H^2}{3}(3R - H)$$

where  $H$  is the depth of water inside the tank and  $R = \sqrt{(A + B)}$  is the radius of the tank. [The constants  $A$  and  $B$  are determined by last two digits of your student's ID. For example: if the student ID is 123456.  $A = 6$  and  $B = 5$ . However, if your last two digits are **00** or **01** or **10**, use  $A = 2$  and  $B = 3$ ].

- a. By using **TWO (2)** numerical methods, determine the depth of the tank during the overflow. Repeat the iteration until the error of successive iteration is less than 5%. Discuss your answers by comparing with the true percent relative error ( $\varepsilon_t$ ). Select the best numerical method and justify your answer.
- b. Derive and simplify the fourth order Lagrange interpolation polynomial for the volume of water inside the tank corresponding to its depth. Subsequently, estimate the volume of the tank when its depth is equivalent to the radius of the tank. Discuss your answer by comparing with the true percent relative error ( $\varepsilon_t$ ).
- c. Estimate the rate of volume for water inside the tank with respect to its depth when the depth is equivalent to the radius of the tank with **TWO (2)** different orders of **ONE (1)** numerical method. Calculate the true percent relative error ( $\varepsilon_t$ ) in each estimate. Select the best order and justify your answer.

[50 marks]

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