BMEN30006 Circuits and Systems
Laboratory 6
Neuromuscular Reflex Model

Important Information

- **Video instructions** for this lab can be found on Canvas at: *BMEN30006_2021_SM1 => Assignments => Workshop 6*.
- Groups for this laboratory can be comprised of one or two students. *Students within a group can submit the same solutions but each should submit their solutions individually*. You can work individually if you chose to do so.
  - If you choose to work in a pair, please label both student's names on your submission so we can identify which students are working together.
- Please enter your answer questions in a PDF document and submit on Canvas. The submission link is located on Canvas as: *BMEN30006_2021_SM1 => Assignments => Workshop*.
- Students with Monday, Tuesday, and Wednesday labs must submit their report by **midnight on Friday, May 28th**.

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Introduction

In this laboratory, we will use SIMULINK (part of MATLAB) to model the dynamics of neuromuscular reflex motion. The model is described by a set of ordinary differential equations (ODEs), which are deterministic, continuous in time, lumped, linear, time-invariant models. We will use Laplace transforms to derive the transfer functions from the ODEs and program them into SIMULINK to run simulations.

Groups for this laboratory are to be comprised of **two** students. Students within a group submit the same report.

The lab report will consist of answers to a set of questions. You will generally need to include figures to show your simulation results in your answers to these questions.

Please read through the whole laboratory (and the Simulink information) prior to your lab session.

The **reports should be submitted at the end of the laboratory**. Submission of reports is electronic via LMS. Each student must upload a report that clearly gives the names and student numbers of both group members.
However, your answer to Question 1 should be handed in during the laboratory to the demonstrator.

Background and modelling

Measuring and modelling the dynamics of neuromuscular reflex motion can reveal the status of the neuromuscular system. Consider an arm where the upper arm is held in a fixed horizontal position and the forearm is free to move in the vertical plane. The initial angle between the forearm and the upper arm is 135° and at $t = 0$ a weight is placed in the hand. This load produces angular motion $d\theta(t)/dt$ of the forearm about the elbow. The angular motion can be recorded and a mathematical model can be derived to interpret the results. We will examine the neuromuscular reflex in two parts: the muscles, representing the actuator of the model, and the muscle spindle, the sensor of the model.

The limb and muscle model

Figure 1 shows a diagram of the arm representing the initial positioning of the forearm with respect to the upper arm. $M_x(t)$ is the external moment acting on the forearm about the elbow joint and $M(t)$ is the muscular response to the external disturbance. Using Newton's second law we can derive the following equation of motion:

$$M_x(t) - M(t) = J\frac{d^2\theta}{dt^2} \qquad \text{where } J \text{ is the moment of inertia} \qquad (1)$$

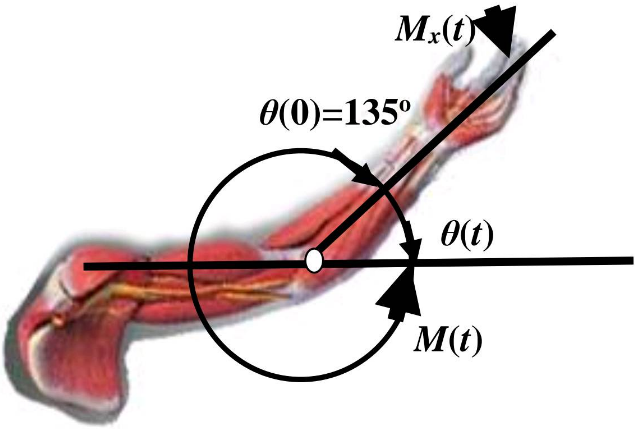


Figure 1. Limb dynamics of the neuromuscular reflex of the arm with forearm held at 135° with respect to the upper arm. $M_x(t)$ is the external moment, $M(t)$ is the muscular response to the disturbance and $\theta(t)$ is the angular displacement.

We will only consider the reflex of the biceps as a single muscle group and the equivalent mechanical analogue circuit of a muscle is represented in Figure 2. $\theta(t)$ and $\theta_1(t)$ are the angular displacements, k is the muscle stiffness and B is the viscous damping parameter. The equations of motion for the muscle model are:

$$M(t) = k(\theta(t) - \theta_1(t)) \qquad (2)$$

and

$$M(t) = M_0(t) + B\frac{d\theta_1(t)}{dt} \qquad \text{where } M_0(t) \text{ isometric torque} \qquad (3)$$

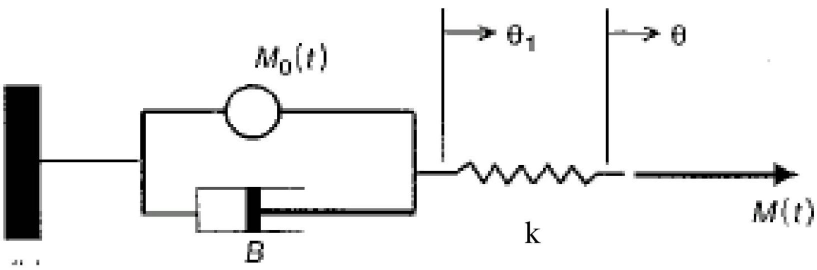


Figure 2. Mechanical analogue of the muscle model.

Isometric torque is when the muscles are contracting (doing work) but produce no displacement. To derive the ODE for the above muscle model, we combine equations (2) and (3) to eliminate θ_1:

From (2): $\theta_1(t) = \theta(t) - \frac{M(t)}{k}$

Differentiate $\frac{d\theta_1(t)}{dt} = \frac{d\theta(t)}{dt} - \frac{1}{k}\frac{dM(t)}{dt}$ (3a)

Eliminate $\theta_1(t)$ in equation (3)

$$M(t) = M_0(t) + B\left[\frac{d\theta(t)}{dt} - \frac{1}{k}\frac{dM(t)}{dt}\right] \qquad (3b)$$

Insert (3b) in equation (1) and re-arrange

$$M_x(t) - M_0(t) = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} - \frac{B}{k}\frac{dM(t)}{dt} \qquad (3c)$$

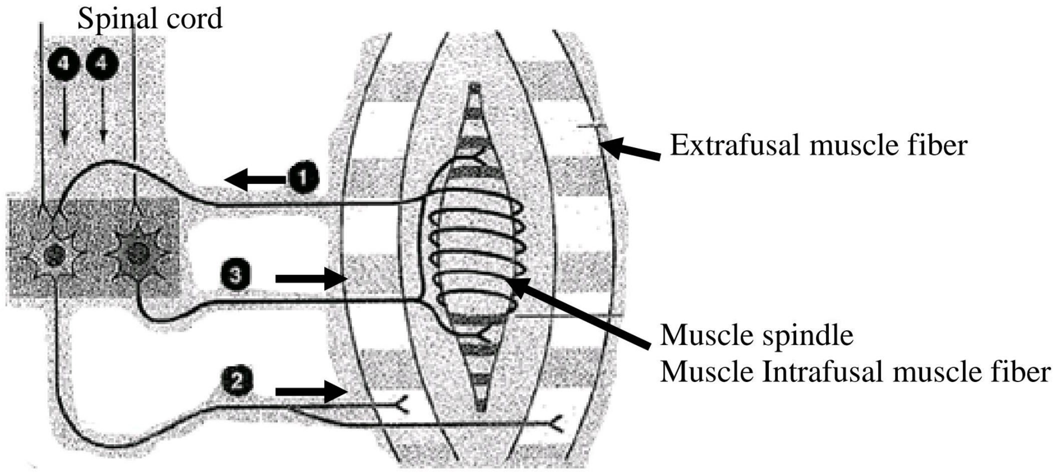
Also differentiate equation (1) with $M_x(t) = 0$

$$-\frac{dM(t)}{dt} = J\frac{d^3\theta(t)}{dt^3} \qquad (3d)$$

To obtain the equation of motion that characterizes the dynamics of the arm, insert equation (3d) into (3c). Angular displacement, $\theta(t)$, responds to the torque exerted by the external disturbance, $M_x(t)$, with the resulting muscular response:

$$M_x(t) - M_0(t) = \frac{BJ}{k}\frac{d^3\theta(t)}{dt^3} + J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} \qquad (4)$$

The muscle spindle model



1. Afferent input from sensory endings of muscle spindle fiber
2. Alpha motor neuron output to regulate muscle fiber
3. Gamma motor neuron output to contractile end of spindle fiber
4. Descending pathways co-activating alpha and gamma motor neurons

Figure 3. Pathways involved in the stretch reflex and co-activation of alpha and gamma motor neurons of a muscle spindle. The 1 to 2 pathway is the stretch reflex pathway.

Angular displacement response to $M_x(t)$ is described by equation (4) and with $M_0(t) = 0$ the equation represents the open loop response to $M_x(t)$. In order to close the loop so that we satisfy equation (4) we need to consider $M_0(t)$, the muscle spindle dynamics. The muscle spindle of the biceps is illustrated in Figure 3 and detects changes in $\theta(t)$. The muscle spindle sends signals to the spinal chord which then sends signals to the contractile muscles to generate $M_0(t)$.

The muscle spindle model is shown in Figure 4 and the neural output of the spindle is proportional to the amount by which it is stretched:

$$M_0(t) = \beta(\theta(t) - \theta_2(\text{t})) \qquad \text{where } \beta \text{ is the gain} \tag{5}$$

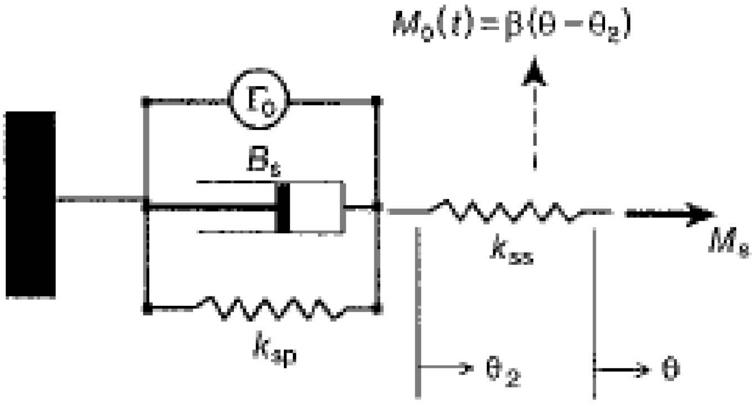


Figure 4. The muscle spindle model.

The elastic stiffness and viscosity of the spindle are represented by k_{sp} and B_s, respectively, of the end regions of the spindle. K_{ss} is the elastic stiffness of the spindle body. Γ_0 represents the contractile part of the spindle, which allows the spindle to be reset via Gamma neuron activation. We assume Γ_0 is constant so that this parameter does not play a role in the dynamics. Therefore, the dynamics of the spindle can be characterized by:

$$M_s(t) = K_{ss}(q(t) - q_2(\text{t})) \tag{6}$$

and

$$M_s(t) = B_{ss}\frac{dq(t)}{dt} + k_{sp}q_2(\text{t}) \tag{7}$$

There is a time delay, T_d, that allows for neural transmission times as well as the delay taken for the muscles to generate force. Eliminating M_s and θ_2 from equations (5) through (7), we derive the equation for the stretch reflex model:

$$\frac{M_0(t)}{\tau} + \frac{dM_0(t)}{dt} = \beta\left[\frac{d\theta(t-T_d)}{dt} + \frac{\theta(t-T_d)}{\eta\tau}\right] \tag{8}$$

where

$$\tau = \frac{B_s}{k_{ss} + k_{sp}} \tag{9}$$

$$\eta = \frac{k_{ss} + k_{sp}}{k_{ss}} \tag{10}$$

Modelling the Neuromuscular Reflex

A physiological representation of a neuromuscular reflex is shown in Figure 5 and the block diagram that models this reflex is presented as open loop sub-systems in Figure 6a and 6b and as one closed loop system in Figure 6c.

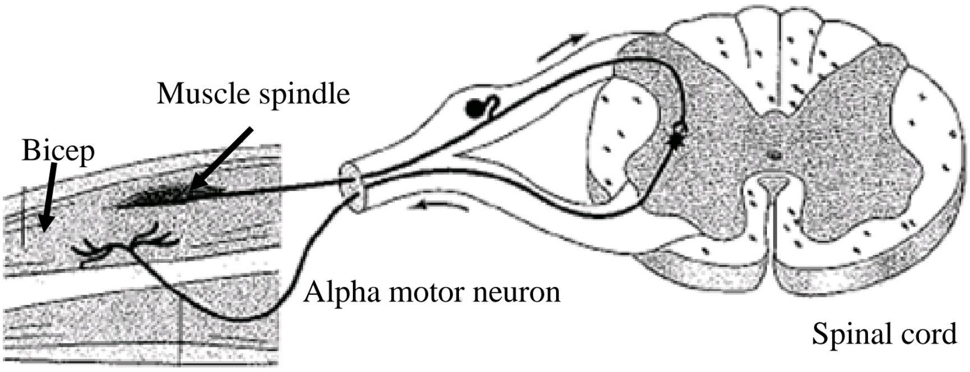


Figure 5. The neuromuscular reflex diagram shows the closed loop of the muscle, the muscle spindle sensing displacement, afferent motor neuron and the Alpha motor neuron to activate the muscle.

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Open Loop System 1

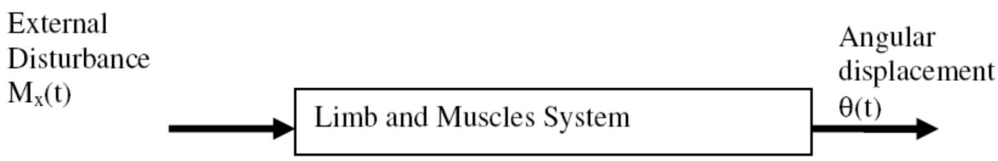

Figure 6a. The system diagram of the limb and muscle dynamics.

The block in Figure 6a represent the transfer functions of the equation characterizing the limb and muscles system. Examining the limb and muscles system on its own provides an open loop response to the reflex with the external disturbance ($M_x(t)$) as the input and angular displacement as the output. The open loop representation of equation (4) is

$$M_x(t) = \frac{BJ}{k}\frac{d^3\theta(t)}{dt^3} + \mathrm{J}\frac{d^2\theta(t)}{dt^2} + \mathrm{B}\frac{d\theta(t)}{dt} \qquad (11)$$

note that the last term is in angular velocity

and to obtain angular displacement we integrate

$$\int M_x(t)\,dt = \frac{BJ}{k}\frac{d^2\theta(t)}{dt^2} + \mathrm{J}\frac{d\theta(t)}{dt} + \mathrm{B}\theta(t) \qquad (12)$$

Open Loop System 2

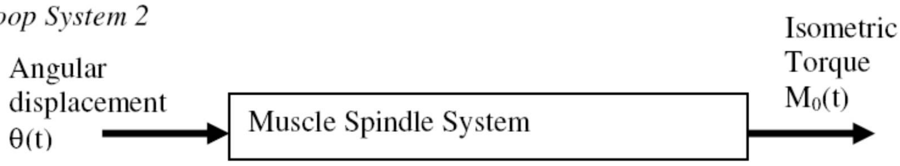

Figure 6b. The system diagram of the muscle spindle dynamics.

The muscle spindle provides the negative feedback for the neuromuscular reflex where the input to the muscle spindle system is angular displacement and the output is isometric torque (Figure 6b), $M_0(t)$.

Close Loop System

Re-arrange equation (8):

$$M_0(t) + \frac{\tau \, dM_0(t)}{dt} = \beta \left[\frac{\tau \, d\theta(t - T_d)}{dt} + \frac{\theta(t - T_d)}{\eta} \right] \qquad (13)$$

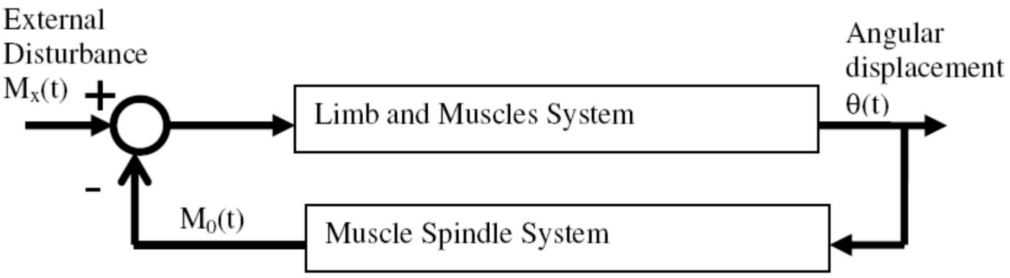

Figure 6c. The neuromuscular reflex closed loop control diagram.

SIMULINK simulations

SIMULINK is a widely used software package in academia and industry for modelling and simulating dynamical systems. You can familiarise yourself with SIMULINK by doing the following exercise:

Type the following in the MATLAB command window:

```
>> simulink
```

A SIMULINK window will appear and in the Help menu select SIMULINK HELP. Read the "Getting Started" section.

In this laboratory, you will build a model of the neuromuscular reflex, run simulations and analyse the responses. The model will be developed from the **bsm23muscles.mdl** file, which you need to download from LMS. This file provides input sources and output sinks for the models, converts radians to degrees (and sets the initial condition of theta, theta(0) = 135), but does not contain the transfer functions required for simulation. The transfer functions will be added as sub-systems with the limb and muscles transfer functions added first so that you can obtain some open loop responses. The addition of the muscle spindle transfer function to the file will complete the model of a neuromuscular reflex.

Limb and muscle model

Before we build the limb and muscles model, we need to derive its Laplace transform. The Laplace transform of equation (12) is in the form $\mathcal{L}\left\{\frac{df}{dt}\right\} = s\mathcal{L}\left\{f\right\} - f(0)$ and the initial conditions have been set in the SIMULINK model. In this case, the Laplace transform of equation (12), where $f(0) = f'(0) = f''(0) = 0$, is:

$$\frac{M_x(s)}{s} = \left[\frac{BJ}{k}s^2 + \mathrm{Js} + \mathrm{B}\right]\theta(s) \qquad (14)$$

Question (1)

Show that the inverse Laplace transform of equation (14) gives you the ODE for limb and muscles system (equation (11)).

Open the **bsm23muscles.mdl** file and insert the above transfer function as shown in figure 7.

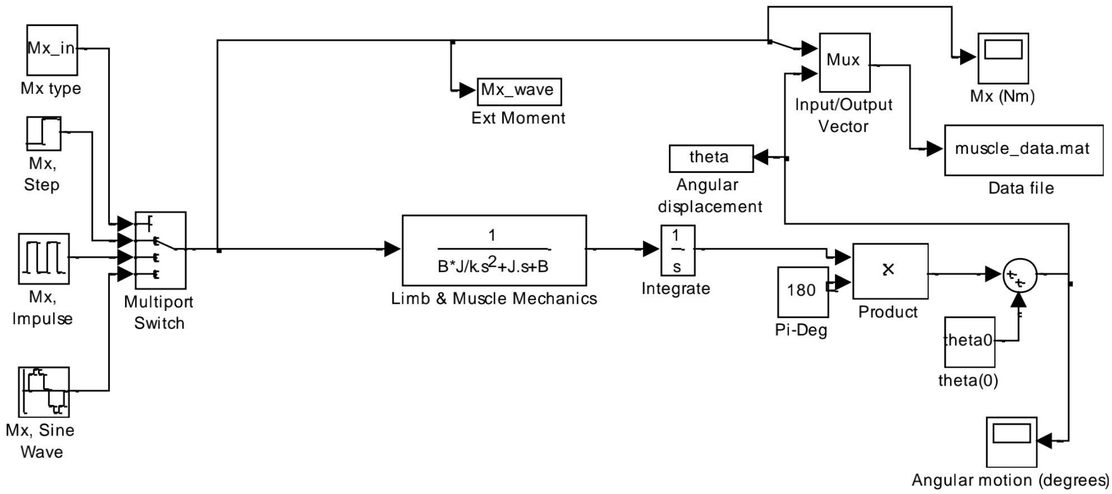

Figure 7. The SIMULINK model of the limb and muscle system.

Insert the boxes by going to:

- View and select Show Library Browser.
- In the SIMULINK Library Browser select Continuous
  - Then drag the Transfer Function box onto your model.
  - Also drag the Integrator box onto your model.
- Wire the model as illustrated and double click on the Transfer Function box and enter the variables *B*, *J* and *k* as per equation (14). You will need to enter `[B*J/k J B]`. If you cannot see the equation as in Figure 7, just increase the size of the box.
- Save this model.

In order to run the simulation, download and open the file **bsm23ini.m**. This file is the script that defines all variables and sets the initial values (Appendix A).

Mx_in is the variable that selects the type of input to excite the simulation:
Mx_in = 1 is a step input (initial input),
Mx_in = 2 is an impulse input, and
Mx_in = 3 is a sinusoidal input.

Run the script by typing the following in the MATLAB command window, or by clicking on the Run icon, in order to set up the variables:

```
>> bsm23ini
```

Run the simulation in SIMULINK. This initial run is with **Mx = 5 Nm**. Examine the plots of **Mx** and **Angular motion**. It may help to right-click on the plots and select Autoscale.

Now plot the responses with **Mx = 2 Nm** and **Mx = 0.5** Nm.

Repeat this with an impulse input (**Mx_in = 2**) and then a sinusoidal input (**Mx_in = 3**).

With modelling, we can examine the dynamics of the biceps on their own and this is something that can NOT be done on subjects/patients.

Question (2)
One can rise and extend the arm until it is level with the shoulder. With the palm facing upwards, extend your forearm as far back as possible (refer to Figure 1). This is the safe limit of the angular displacement that the limb can achieve around the elbow. What is the limit of angular displacement? This limit will assist in interpreting the responses of your simulations.

Question (3)
Examine the plots and find the maximum value of external moment (Mx) required to always be within the safe range of angular displacement for each input type. Include in your answer example plots of the angular displacement for each case.

Question (4)
For each case in Question (3), describe the effect of each type of input (step, impulse, sinusoidal) and the reason for each response in terms of the resulting angular displacement.

Modelling a neuromuscular reflex

To complete the model of neuromuscular reflex, we add the muscle spindle system to the limb and muscles model. We need to derive the Laplace transform of the muscle spindle. The Laplace transform of equation (13) with zero initial conditions is:

$$M_0(s) = \beta \left[\frac{\tau s + \frac{1}{\eta}}{\tau s + 1} \right] e^{-sTd} \theta(s) \qquad (15)$$

Add the following to your model, as per figure 8:

- In the SIMULINK Library Browser select Continuous
- First drag **Transport Delay** box onto your model
  - Wire the model as illustrated and double click on the box and enter the variable Td.
- Then drag the **Transfer Function** box onto your model.
  - Wire the model as illustrated and double click on the Transfer Function box and enter the variables **tau** and **eta** as per Laplace transform of the muscle spindle.
- In the SIMULINK Library Browser select **Math Operations**
- Select and drag the **Gain box** onto your model.
  - Wire the model as illustrated and double click on the box and enter the variable beta.
- Select and drag the **subtract box** onto your model.
  - Wire the model as illustrated and rename Sum.
- In the SIMULINK Library Browser select **Sinks**, select **To Workspace** and wire as shown
  - Double click and enter the variable M0
- Save this model.

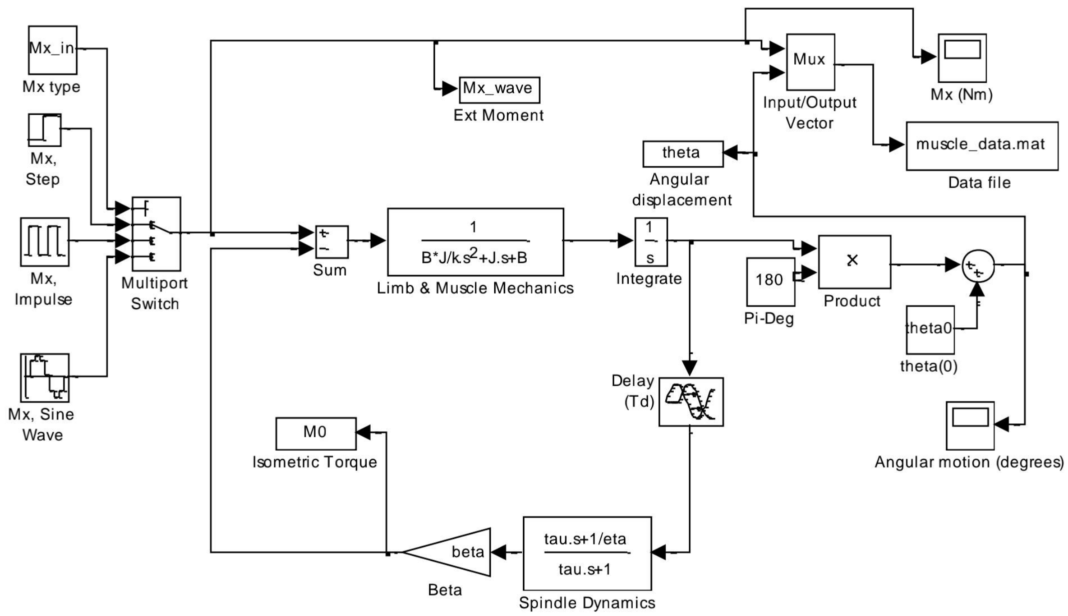

Figure 8. The SIMULINK model of a neuromuscular reflex.

Reset to **Mx=5** and **Mx_in=1** in bsmini.m. Run the initialisation script again:

```
>> bsm23ini
```

Run the simulation for the following conditions:

1. Input type is a **step function** (Mx_in=1): This initial run is with a) **Mx=5 Nm**.
   Find the maximum value of **Mx** to always be within the safe range of angular displacement. Plot this response.

2. Change the input type to an **impulse function** (Mx_in=2). Find the maximum value of **Mx** to always be within the safe range of angular displacement. Plot this response.

3. Change the input type to a **sinusoidal function** (Mx_in=3). Find the maximum value of **Mx** to always be within the safe range of angular displacement. Plot this response.

Question (5)
Give all of your maximum values for **Mx** that you obtained along with plots of the angular displacement.

Question (6)
For each case in Question (5), describe the effect of each type of input (step, impulse, sinusoidal) and the reason for each response in terms of the resulting angular displacement.

Question (7)
Compare the closed loop responses in Question (5) to the open loop responses in Question (3). What differences does introducing feedback have on the responses and why?

Using the parameters you obtained above in Question (5), repeat the simulations with the different types of inputs for the following conditions:

1) no delay time, **Td = 0** (compared to Td = 0.02)

2) feedback gain, **beta = 0** (compared to beta = 100)

3) feedback gain, **beta = 200** (compared to beta = 100).

Question (8)
For each of the cases above (1-3), describe the effect of changing the delay time and feedback gain, and state whether or not the resulting system is stable.

References

Sherwood L. Human Physiology - From Cells to Systems, 5th Edition Brooks/Cole, Belmont, 2004

Khoo M. Physiological control systems: analysis, simulation, and estimation, Published New York : IEEE Press, 2000

The MathWorks, Inc. MATLAB and Simulink. http://www.mathworks.com/

Appendix A: Initialization script for the model of a neuromuscular reflex.

```
% Script file for bsm23ini.m neuromuscular reflex simulation lab
%
%Initial state
% J = moment of inertia
J=0.1
% k = muscle stiffness
k=50.0
% B = viscous damping
B=2
% Td = lags in neural transmission along afferent and efferent + muscle AP into
force
Td=0.02
% tau =  Bs/(kss+ksp) = viscous of spindle/(stiffness of nuclear bag + stiffness of
spindle)
tau=1/300
% beta = amount nuclear bag is stretched
beta=100
% eta = (kss+ksp)/ksp = (stiffness of nuclear bag + stiffness of spindle)/stiffness
of spindle
eta=5
% Moment (Nm)
Mx=5
% theta (deg) initial forearm position
theta0=135
% Mx_in is the type of input: 1=step 2=impulse 3=sine
Mx_in=1
%
%cycles = oscillation frequency (Hz) for sine input
cycle=5
freq=2*(pi)*cycle
%
%open model
open('bsm23muscles.mdl')
```