

c

1.

A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \quad x \geq 0$$

- (a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

- (b) Find the equation of the normal to the curve at P . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(4)

2.

- (a) Show that the equation

$$8 \tan \theta = 3 \cos \theta$$

may be rewritten in the form

$$3 \sin^2 \theta + 8 \sin \theta - 3 = 0$$

(3)

- (b) Hence solve, for $0 \leq x \leq 90^\circ$, the equation

$$8 \tan 2x = 3 \cos 2x$$

giving your answers to 2 decimal places.

(4)

3.

Solutions relying on calculator technology are not acceptable in this question.

(i)

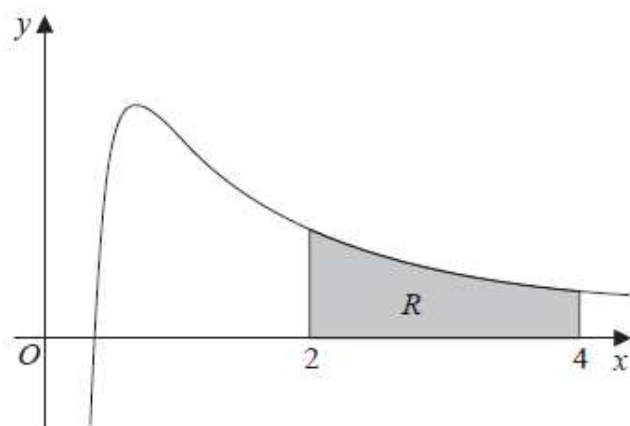


Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \quad x > 0$$

The region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 4$.

Find the exact area of R .

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^6 \left(\frac{1}{2}x^2 + k \right) dx = 55$$

(4)

4.

- (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b \log_2 3$, where a and b are constants to be found.

(4)

- (ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2 \log_5(y + 1) - 1$$

(5)

5.

A curve C has equation

$$y = 4x^3 - 9x + \frac{k}{x} \quad x > 0$$

where k is a constant.

The point P with x coordinate $\frac{1}{2}$ lies on C .

Given that P is a stationary point of C ,

- (a) show that $k = -\frac{3}{2}$ (4)

- (b) Determine the nature of the stationary point at P , justifying your answer. (2)

The curve C has a second stationary point.

- (c) Using algebra, find the x coordinate of this second stationary point. (4)
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6.

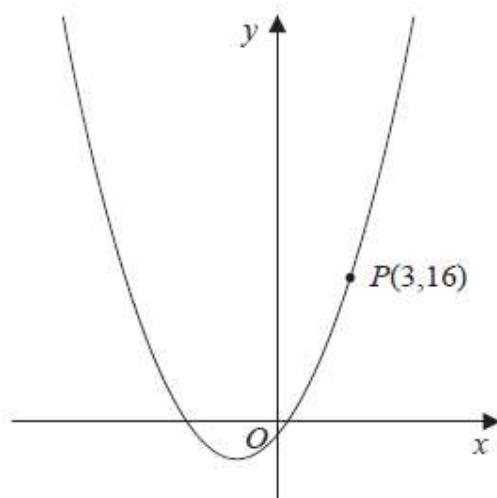


Figure 1

Figure 1 shows part of the curve with equation $y = x^2 + 3x - 2$

The point $P(3, 16)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P .

(2)

The point Q with x coordinate $3 + h$ also lies on the curve.

(b) Find, in terms of h , the gradient of the line PQ . Write your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

Mechanics

7.

[In this question \mathbf{i} and \mathbf{j} are perpendicular horizontal unit vectors.]

A particle P of mass 2 kg moves under the action of two forces, $(2\mathbf{i} + 3\mathbf{j})$ N and $(4\mathbf{i} - 5\mathbf{j})$ N.

(a) Find the magnitude of the acceleration of P .

(4)

At time $t = 0$, P has velocity $(-u\mathbf{i} + u\mathbf{j})$ m s⁻¹, where u is a positive constant.

At time $t = T$ seconds, P has velocity $(10\mathbf{i} + 2\mathbf{j})$ m s⁻¹.

(b) Find

(i) the value of T ,

(ii) the value of u .

(5)

8.

A small ball is projected vertically upwards from a point O with speed 14.7 m s⁻¹. The point O is 2.5 m above the ground. The motion of the ball is modelled as that of a particle moving freely under gravity.

Find

(a) the maximum height above the ground reached by the ball,

(4)

(b) the time taken for the ball to first reach a height of 1 m above the ground,

(4)

(c) the speed of the ball at the instant before it strikes the ground for the first time.

(3)

Statistics

9.

There are 64 girls and 56 boys in a school.

Explain briefly how you could take a random sample of 15 pupils using

(a) a simple random sample, (3)

(b) a stratified sample. (3)

10.

A doctor wishes to test a new treatment for a particular skin condition. She asks patients who come to her surgery suffering from this condition if they are willing to take part in a trial. She observes that the probability of such a patient agreeing to take part is 0.3.

(Assume throughout this question that the behaviour of any patient is independent of the behaviour of all other patients.)

(a) If she asks 25 patients to take part in the trial, find the probability that

(i) four or fewer will agree,

(ii) exactly six will agree. (4 marks)

With the assistance of other doctors, 900 patients are found to take part in the trial. The patients are split randomly into two groups of 450. One group receives the standard treatment and the other group receives the new treatment.

The probability of a patient who receives the new treatment withdrawing before the end of the trial is 0.22.

(b) For each of the following cases state, giving a reason, whether or not the binomial distribution is likely to provide an adequate model for the random variable R .

(i) R is the total number (out of 900) of patients withdrawing before the end of the trial. (2 marks)

(ii) R is the total number of patients asked in order to obtain 900 to take part in the trial. (2 marks)

END