

Advanced Microeconomics, Assignment #2.

This assignment is due on or before Wednesday, December 22 at 5PM.

Please upload a typed document (either word or pdf).

If you work alone, or if you worked in a group but prefer to submit your own version of the assignment, please do not forget to put **your student number** at the top of the first page.

If you worked in a group and agree on the document to submit, please include **the student numbers of all students who worked together** (when relevant) on the front page.

Exercise 1: Adverse selection and monopolistic principal.

An environmental state agency must regulate the emissions emanating from a private national firm. To do so, the agency designs a contract which imposes the emission level $e \geq 0$ that the firm can produce if it pays a fixed tax $t \geq 0$. Emissions are verifiable so that the firm who accepts the contract cannot pollute more what is stipulated in the contract.

The firm has a profit function given by $\pi(e, t) = \theta e - t$, where θ measures the benefits that it gets from being allowed to emit more pollutants. Assume that the firm has either high or low returns from polluting so that we either have $\theta = 80$ or $\theta = 120$.

The firm accepts the contract so long as $\pi(e, t) \geq 100$.

The state agency is utilitarian. Its objective takes into consideration the benefits of collecting the tax revenue, the profits of the firm and the cost of emissions for the society. The amount of taxes that is raised is used to fund public health and education so that €1 tax generates an overall value of €2. Specifically, the objective function of the state agency is given by

$$B(e, t) = 2t + \pi(e, t) - 4e^2,$$

The first component ($2t$) reflects the fact that a payment of €1 tax by the firm generates a return of €2, the second component captures the profits of the firm and the last component ($4e^2$) represents the cost of emissions for society.

- i. Consider an environment where the state agency can perfectly verify the value of θ . What would be the optimal levels of taxes that the firm would pay to be granted the right to emit $e = 40$?
- ii. Find the optimal contracts (taxes and emission levels) that the state agency would propose to the firm when it can verify the true value of θ .
- iii. Explain what would happen if the contracts you found in the previous question were used when the state agency cannot verify the true value of θ . (Prove your answer.)

- iv. Solve for the optimal taxes and emission levels that should apply when the state agency cannot verify the true value of θ and when it knows that $\theta = 80$ with a probability of $1/2$. (Feel free to use some shortcuts relying on constraints that bind.)
- v. Comment on the contracts you found comparing the expected level of emissions and the expected level of taxes that are raised when information is symmetric and when it is not.
Explain, in a short paragraph, whether you think that your conclusions are specific to the model or whether they would arise in a more general setting.

Exercise 2: Adverse selection, competing principals and signalling.

Consider a setting where several agents are facing competing employers. Agents have distinct abilities. Let $a \in \{2,4\}$ denote an agent's ability. The variable a is known to the agent only and is not verifiable.

Principals know that, a priori, $a = 2$ with a probability of $1/2$.

Agents have the possibility to select an education level $s \in [0,1]$. Note that the level of education is a continuous variable. The level of education chosen by any agent is verifiable. Employers can therefore offer different wages based on the level of education chosen by the agent. Let $w(s)$ denote the wage paid to a agent with education level s .

Let $q(s)$ be the probability that an employer assigns to facing a high ability agent, that is such that $a = 4$, when the level of education chosen by the agent is s :

$$q(s) = \Pr(a = 4/s).$$

All parties are risk neutral. The payoff to a principal is given by $\pi = a - w$.

The utility of an agent depends on his ability. The high ability agent has a lower cost of education:

$$u_a(w, s) = w - \frac{8}{a}s.$$

Utility of a high ability agent who has a productivity $a = 4$

$$u_4(w, s) = w - 2s.$$

Utility of a low ability agent who has a productivity $a = 2$

$$u_2(w, s) = w - 4s.$$

TASK #1: Find a separating equilibrium whereby only high ability agents select a positive education level: $s(2) = 0$ and that $s(4) = s^* > 0$.

Consider the following beliefs: $q(s) = 0$ for all $s \in [0, s^*[$ and $q(s) = 1$ for all $s \in [s^*, 1]$. This means that any agent with an education level below s^* is assumed to have an ability $a = 2$ while any agent with a level of education at or above s^* is assumed to have an ability $a = 4$.

- i. Given the employers' beliefs, what value does the competitive wage $w(s)$ take for any given value $s \in [0, 1]$?
- ii. Find all values for s^* such that the agents education strategies, whereby only the high ability agents choose $s = s^*$, form an equilibrium given the beliefs triggered and given your answer to i..

TASK #2: Show that a pooling equilibrium where no one gets educated (so that $s(4) = s(2) = 0$) cannot exist.

Assume that there exists such a pooling equilibrium where no one gets educated. In such an equilibrium we must have $q(0) = \frac{1}{2}$ since all agents choose $s = 0$. Therefore, observing that $s = 0$ has no impact on the probability of facing a high or low ability agent. Moreover, since agents are risk neutral and employers must get zero expected payoffs, the wage paid in equilibrium must be such that

$$w(0) = \frac{1}{2}(2) + \frac{1}{2}(4) = 3.$$

Given this information, prove that there exists an education level $\hat{s} > 0$ and a wage $w(\hat{s})$ such that both of the points below hold

- High ability agents would be better-off selecting $s = \hat{s}$ while low ability agents are better-off selecting $s = 0$.
- Any employer offering $w(\hat{s})$ would make money out of the high ability agents that are the only ones he attracts.