University of Manchester

ECON61001: Econometric Methods

Mid-Term Exam

Release date/time: 10/11/21, 16.00hrs Greenwich Mean Time (GMT)

Submission deadline: 12/11/21, 16.00hrs GMT

Instructions:

- You must answer **all three** questions.
- Your answers could be typed or hand-written (and scanned to a single pdf file that can be submitted) or a combination of a typed answer with included images of algebra or figures. **The name of this file must be your student registration number.**
- Where relevant, questions include word limits. These are limits, not targets. Excellent answers can be shorter than the word limit. If you go beyond the word limit the additional text will be ignored. Where a question includes a word limit you HAVE to include a word count for your answer (excluding formulae). You could use https://wordcounter.net to obtain word counts.
- Candidates are advised that the examiners attach considerable importance to the clarity with which answers are expressed.
- **You must correctly enter your registration number and the course code on your answer.**

1. An economist wishes to analyze how house prices depend on certain characteristics of the house in a city in the US. To this end, she takes a random sample from house sales in the city in a particular month and collects the following information: $price$, the house price in thousands of dollars; $bdrms$, the number of bedrooms in the house; $sqrft$, the size of the house in square feet. The sample size is $N = 88$. Based on this sample, she estimates the regression model below in R,

$$\log(price_i) = x_i'\beta_0 + u_i, \tag{1}$$

where $x_i' = [1, bdrms, \log(sqrft_i)]$, $log(.)$ denotes the natural logarithm of the variable in parentheses, and $\beta_0 = (\beta_{0,1}, \beta_{0,2}, \beta_{0,3})'$, obtaining the following output:

```
Coefficients:Estimate Std. Error
(Intercept) -0.62340    0.69758
bdrms        0.03811    0.03034
lsqrft       0.80825    0.09869

Multiple R-squared:  0.5611
```

where `lsqrft` denotes $\log(sqrft)$ You may assume that (1) is the true model for $log(price)$ and that the data satisfies Assumptions CS2-CS5 in lectures.

(a) Test whether the explanatory variables collectively help to explain the log of house prices. Be sure to explain all aspects of your test including the null and alternative hypotheses in terms of restrictions on β_0, how your test statistic is calculated, the decision rule and the outcome of the test. From the conventional choices of significance levels (10%, 5%, 1%), choose the one that gives the most informative answer. **[8 marks]**

(b) Test whether the number of bedrooms has a positive impact on the house price. Be sure to explain all aspects of your test including the null and alternative hypotheses expressed in terms of restrictions on β_0, how your test statistic is calculated, the decision rule and the outcome of the test. From the conventional choices of significance levels (10%, 5%, 1%), choose the one that gives the most informative answer. **[6 marks]**

(c) What is the interpretation of $\beta_{0,3}$? Justify your answer. **[6 marks]**

(d) The economist wishes to use the regression results to calculate the impact of an increase in $sqrft$ by 1 on the price for a 1000 square foot house holding all other factors equal, and asks your help. Provide the answer for her: report the figure to 4 decimal places and be sure to explain your calculation. **[5 marks]**

Continued over

1. (e) The economist then asks for your help in calculating the impact of an increase in $sqrft$ by 200 on the price for a 1000 square foot house holding all other factors equal. Provide the answer for her: report the figure to 4 decimal places and be sure to explain your calculation. **[8 marks]**

2. Consider the linear regression model,

$$y \; = \; X\beta_0 \; + \; u,$$

that satisfies the conditions in the Classical Assumptions CA1-CA6 from lectures. Let $\hat{\beta}_T$ and $\hat{\sigma}_T^2$ denote the OLS estimators of β_0 and σ_0^2 where $Var[u_t] = \sigma_0^2$ and u_t denotes the t^{th} element of u. Define $X = (X_1, X_2)$ where X is $T \times k$ and X_ℓ is $T \times k_\ell$ for $\ell = 1, 2$ and $\hat{\beta}_T$ and β_0 are partitioned conformably as $\hat{\beta}_T = (\hat{\beta}'_{T,1}, \hat{\beta}'_{T,2})'$ and $\beta_0 = (\beta'_{0,1}, \beta'_{0,2})'$ where $\hat{\beta}_{T,\ell}$ and $\beta_{0,\ell}$ are $k_\ell \times 1$ for $\ell = 1, 2$. Define $M_\ell = I_T - P_\ell$ where $P_\ell = X_\ell(X'_\ell X_\ell)^{-1}X'_\ell$ for $\ell = 1, 2$.

Suppose it is desired to test $H_0 \; : \; R_1\beta_{0,1} \; = \; r_1$ versus $H_1 \; : \; R_1\beta_{0,1} \; \neq \; r$ where R_1 is an $n_r \times k$ matrix of specified constants, and $rank(R_1) = n_r$. Consider the following decision rule:

- *Decision rule: reject $H_0 : \; R_1\beta_{0,1} = r$ at the $100\alpha\%$ significance level if $F_1 > F_{n_r,T-k}(1-\alpha)$, where*

$$F_1 \; = \; \frac{(R_1\hat{\beta}_{T,1} \; - \; r)'[R_1(X'M_2X_1)^{-1}R'_1]^{-1}(R_1\hat{\beta}_1 \; - \; r)}{n_r\hat{\sigma}_T^2},$$

  and $F_{n_r,T-k}(1-\alpha)$ is the $100(1-\alpha)^{th}$ percentile of the F - distribution with $(n_r, T-k)$ degrees of freedom,

Show that if this decision rule is used then the probability of a Type I error is α. **[17 marks]**

Hint: you may quote from the Lecture Notes both the generic formula for the F - statistic for testing $H_0 : \; R\beta_0 = r$ and its distribution under H_0.

Continued over

3. Consider the simple linear regression model,

$$y_i \;=\; x_i'\beta_0 \,+\, u_i, \tag{2}$$

where $x_i' = (1, m_i)$, m_i is a scalar, and $\beta_0 = (\beta_{0,1}, \beta_{0,2})'$. Suppose that (2) represents the true model for y_i, $\{(m_i, u_i)\}_{i=1}^N$ is a sequence of independently and identically distributed (i.i.d.) random vectors with $E[u_i|x_i] = 0$, $Var[u_i|x_i] = \sigma_0^2$ and $E[x_i x_i'] = Q$, a positive definite matrix of finite constants. The OLS estimator of the slope coefficient can be written as:

$$\hat{\beta}_{N,2} \,-\, \beta_{0,2} \;=\; \frac{\sum_{i=1}^N (m_i \,-\, \bar{m})u_i}{\sum_{i=1}^N (m_i \,-\, \bar{m})^2}. \tag{3}$$

(a) Prove the following results as $N \to \infty$, being sure to justify your answer carefully in each case.

(i) $N^{-1}\sum_{i=1}^N (m_i \,-\, \bar{m})u_i \stackrel{p}{\to} 0$; **[9 marks]**

(ii) $N^{-1}\sum_{i=1}^N (m_i \,-\, \bar{m})^2 \stackrel{p}{\to} \sigma_m^2$, where $\sigma_m^2 = Var[m_i]$; **[6 marks]**

(iii) $N^{-1/2}\sum_{i=1}^N (m_i \,-\, \bar{m})u_i \stackrel{d}{\to} N\,(\,0,\, \sigma_0^2\sigma_m^2\,)$. **[20 marks]**

(b) Using the results in part (a), characterize the limiting behaviour of the following statistics as $N \to \infty$. Be sure to justify your answer carefully in each case:

(i) $\hat{\beta}_{N,2}$; **[5 marks]**

(ii) $N^{1/2}(\hat{\beta}_{N,2} \,-\, \beta_{0,2})$. **[6 marks]**

Hint for parts (a) & (b): for any i.i.d. random variable/vector z_i:

- *you can quote the Weak Law of Large Numbers $N^{-1}\sum_{i=1}^N z_i \stackrel{p}{\to} \mu_z$ but must derive μ_z for the particular choice of z_i;*
- *you can quote the Central Limit Theorem $N^{-1/2}\sum_{i=1}^N (z_i \,-\, \mu_z) \stackrel{d}{\to} N(0, \Omega_z)$ but must derive μ_z and Ω_z for the particular choice of z_i.*

(c) What is the behaviour of $N(\hat{\beta}_{N,2} - \beta_{0,2})$ as $N \to \infty$? Provide a brief heuristic justification for your answer but no formal derivations are required. **[4 marks]**

END OF EXAMINATION

1 Table 1: Percentage Points for the t distribution

Student's t Distribution Function for Selected Probabilities
The table provides values of $t_{\alpha,v}$ where $\Pr(T \leq t_{\alpha,v}) = \alpha$ and $T \sim t_v$

α	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9975	0.999	0.9995
ν	Values of $t_{\alpha,v}$									
1	1.000	1.376	3.078	6.314	12.706	31.821	63.657			
2	0.816	1.061	1.886	2.920	4.303	6.965	9.925			
3	0.765	0.978	1.638	2.353	3.182	4.541	5.841			
4	0.741	0.941	1.533	2.132	2.776	3.747	4.604			
5	0.727	0.920	1.476	2.015	2.571	3.365	4.032	4.773		
6	0.718	0.906	1.440	1.943	2.447	3.143	3.707	4.317	5.208	
7	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.678	0.847	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.678	0.846	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.677	0.846	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.677	0.845	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
110	0.677	0.845	1.289	1.659	1.982	2.361	2.621	2.865	3.166	3.381
120	0.677	0.845	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	0.842	1.282	1.645	1.960	2.326	2.576	2.808	3.090	3.297

2 Table 2: Percentage Points for the χ^2 distribution

The χ^2 Distribution Function for Selected Probabilities

The table provides values of $\chi^2_{\alpha,v}$ where $\Pr(\chi^2 \leq \chi^2_{\alpha,v}) = \alpha$ and $\chi^2 \sim \chi^2_v$

α	0.005	0.01	0.025	0.05	0.1	0.5	0.9	0.95	0.975	0.99	0.995
v	Values of $\chi^2_{\alpha,v}$										
1	0.000	0.000	0.001	0.004	0.016	0.455	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	1.386	4.605	5.991	7.378	9.210	10.60
3	0.072	0.115	0.216	0.352	0.584	2.366	6.251	7.815	9.348	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	3.357	7.779	9.488	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.610	4.351	9.236	11.07	12.83	15.09	16.75
6	0.676	0.872	1.237	1.635	2.204	5.348	10.64	12.59	14.45	16.81	18.55
7	0.989	1.239	1.690	2.167	2.833	6.346	12.02	14.07	16.01	18.48	20.28
8	1.344	1.646	2.180	2.733	3.490	7.344	13.36	15.51	17.53	20.09	21.95
9	1.735	2.088	2.700	3.325	4.168	8.343	14.68	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	4.865	9.342	15.99	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	5.578	10.34	17.28	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	6.304	11.34	18.55	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	7.042	12.34	19.81	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	7.790	13.34	21.06	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	8.547	14.34	22.31	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	9.312	15.34	23.54	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	10.86	17.34	25.99	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.034	8.897	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.643	9.542	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.260	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.886	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
35	17.19	18.51	20.57	22.47	24.80	34.34	46.06	49.80	53.20	57.34	60.27
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
45	24.31	25.90	28.37	30.61	33.35	44.34	57.51	61.66	65.41	69.96	73.17
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	89.33	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	99.33	118.5	124.3	129.6	135.8	140.2
150	109.1	112.7	118.0	122.7	128.3	149.3	172.6	179.6	185.8	193.2	198.4
200	152.2	156.4	162.7	168.3	174.8	199.3	226.0	234.0	241.1	249.4	255.3

3 Table 3: Upper 5% percentage points for the F distribution

The F Distribution Function for $\alpha = 0.05$												
The table provides values of F_{α,v_1,v_2} where $\Pr(F \geq F_{\alpha,v_1,v_2}) = 0.05$ and $F \sim F(v_1, v_2)$												
	$v_1 \rightarrow$											
$v_2 \downarrow$	1	2	3	4	5	6	7	8	9	10	12	15
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.96
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05	1.97	1.89
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.87
55	4.02	3.16	2.77	2.54	2.38	2.27	2.18	2.11	2.06	2.01	1.93	1.85
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.81
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.79
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.78
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.77
110	3.93	3.08	2.69	2.45	2.30	2.18	2.09	2.02	1.97	1.92	1.84	1.76
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.73

4 Table 4: Upper 1% percentage points for the F distribution

The F Distribution Function for $\alpha = 0.01$												
The table provides values of F_{α,v_1,v_2} where $\Pr(F \geq F_{\alpha,v_1,v_2}) = 0.01$ and $F \sim F(v_1, v_2)$												
	$v_1 \rightarrow$											
$v_2 \downarrow$	1	2	3	4	5	6	7	8	9	10	12	15
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.60
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74	2.61	2.46
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.42
55	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66	2.53	2.38
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.45	2.31
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.27
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.39	2.24
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.22
110	6.87	4.80	3.96	3.49	3.19	2.97	2.81	2.68	2.57	2.49	2.35	2.21
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.31	2.16