SM4332 – OPERATIONS RESEARCH I

ASSIGNMENT 1

"INDUSTRIAL OPTIMISATION PROBLEM"

(Due 1 November 2021)

Background

Carbonate is a major carbon fibre producer in **Kyoto**. The new CEO wants to implement a new environmental policy by committing itself to reducing its annual emission of pollutants (**Table 1**). The company will invest in upgrading its technology regarding two aspects: better filters and better fuels. The financial team has evaluated the annual cost of the investment depending on the percentage of filters and fuels upgraded to the new technology (**Table 2**). In addition, the engineering team has provided you with data estimating the reduction of pollutants for various percentages of upgrade (**Table 3**). The company has hired you to find out the percentage of its filters and fuels that must be upgraded in order to attain the desired reduction in annual emissions while minimising the total cost of the upgrade.

Question 1: The first step in the analysis is to visualise and understand the data in **Table 2** and **Table 3** in detail. For each column in **Table 2** and **Table 3** (i.e. 8 columns in total), fit a linear model using *Microsoft Excel* or *R*.

Question 2: Using the linear models from Q1 and the data in **Table 1**, formulate the problem as a linear programming (LP) problem. The objective is to minimise the total cost, which is a function of the percentage of fuels and filters upgraded (i.e., the linear models built from the data in **Table 2**). In addition, there will be one constraint per pollutant. Each constraint models the requirement that the total reduction of pollutant emission achieved (which is a function of the percentage of upgrade of each technology, that is, the linear models built from the data in **Table 3**) is larger than or equal to the target reduction in **Table 1**. Remember that the decision variables are also constrained. Include the LP definition in your report and label clearly the objective, constraints and decision variables.

Question 3: Solve the LP problem from Q2 graphically, i.e. draw a coordinate system with all the constraints, indicate where the feasible region is, and draw the objective function going through the optimal solution. Provide the coordinates of the optimal solution.

Question 4: Two other types of technologies are also considered for upgrade: Furnaces and smokestacks. Data regarding the financial cost and pollution reduction is given directly as a linear factor (i.e., the slope of the linear function) of the upgrade ratio (i.e., a value of 0 corresponds to no upgrade and a value of 1 to 100% upgrade) in **Table 4**. Extend the Linear Programming (LP) model from Q3 to include these two additional decision variables. Note that the inclusion of these two decision variables will require you to modify both the objective and the constraints. Solve the resulting LP problem. What is the percentage of filters, fuels, furnaces and smokestacks that should be upgraded to the new technologies? Compare your results to Q3 and analyse the reason(s) for a potential shift in the optimal solution (compared to Q3).

Table 1: Target reductions for each type of pollutant

Pollutant	Target reduction in annual emission (tons × 1000)
Particulate Matter (PM)	10
Sulfur Dioxied (SO2)	34
Carbon Monoxide (CO)	50

Table 2: Annual cost of investment as a percentage of technological upgrade

% Upgrade	Annual cost (£M)	
	Filters	**Fuels**
10	5	10
20	5	9
45	8	12
65	10	11
85	13	13
90	20	14

Table 3: Annual reduction in pollutants for a given percentage of technological upgrade

% Upgrade	Reduction in annual emission (tons x 1000)					
	Filters			**Fuels**		
	PM	**SO2**	**CO**	**PM**	**SO2**	**CO**
10	12	35	20	8	10	2
20	15	20	22	10	15	10
45	20	15	15	13	17	15
65	30	18	18	14	18	19
85	25	30	30	13	20	25
90	27	35	45	17	21	35

Table 4: Reduction in emissions and financial cost as a linear factor of the ratio of technological upgrade *e.g. a factor of 0.2 (Smokestacks, PM pollutant) should be interpreted as a reduction in PM pollutant by 0.2 * %Upgrade_Smokestacks, where %Upgrade_Smokestacks is a value between 0 and 100. Similarly, a factor of 0.1 (annual cost, furnaces) should be interpreted as an annual cost of 0.1 * %Upgrade_Furnaces, where %Upgrade_Furnaces is a value between 0 and 100.*

Technology	Reduction in annual emission for each pollutant (tons x 1000)			Annual Cost (£M)
	PM	SO2	CO	
Furnaces	0	0.03	0.1	0.1
Smokestacks	0.20	0.15	0.08	0.05