Course Instructor: Dr. Anubha Gupta

Instructions

1. Submis[illegible] PM
2. [illegible]

Theory Problems:

- Solutions can be submitted as a scanned .pdf file of your written solutions.
- Alternatively, a solution can be prepared in doc/latex as well. For that please export it in .pdf format.
- Add your theory pdf file as A1_Theory Roll_No.pdf.

Programming Problems:

- You can use python/MATLAB for programming problems.
- Along with the main code file, please submit all required dependencies.
- Also add a report (as A1_Prog_ RollNo.pdf) with a brief summary of your coding outputs or generated plots.

Submit an A1_RollNo.zip file on Google classroom with all required files.

Theory questions

1. Assume a cricket game where a player can only score a six or four at each ball. Let, Ben be a cricket player who can score a four at each ball with a probability of 0.4. Find the probability for the following:

   (a) [1 Marks] If Ben takes ten independent trails, let **X** denote an event, **X**: number of sixes scored by Ben. What is the probability distribution of **X**?

   (b) [2 Marks] Find the probability that in ten independent trials Ben will score as mentioned in Table 1.

Table 1: Score at 10 independent trial

Trail No.	1	2	3	4	5	6	7	8	9	10
Score	4	6	6	4	4	6	4	6	6	4

   (c) [2 Marks] Suppose Denial is a player in opposite team who scores four at each ball with a probability of 0.7. Assume that a game consists of eleven balls and in order to win the game a player needs to score more than the other player. What is the probability that Denial will win the game?

(d) [3 Marks] Suppose the match is interrupted by rain. Assume that Ben and Denial has equal probability to win and the first one to win a total of five games is the winner. This happens when Ben has won a games while, Denial has won b games, where, $a < 5$ and $b < 5$. Calculate the probability that if the match was not interrupted, Ben would have been the winner.

2. A magician has three bags number A, B, and C. All the bags contain 5 dices. All the dices have six sides and identical in shape and size. The dice can be fair or rigged (with number 2 mentioned on all the sides). The ratio of fair to rigged dice is mentioned in Table 2:

Table 2: Balls in bag

Bag	A	B	C
Fair dice	2	4	3
Rigged dice	3	1	2

(a) [1 Marks] Suppose the magician takes out a dice from the box, what is the probability that the magician takes out a fair dice?

(b) [1 Marks] Suppose the magician takes out a dice from any of the bag and roll it. What is the probability that the number is 2?

(c) [3 Marks] Suppose the magician takes out a dice from any of the bag and roll it. Let the number appear of the dice be N_1. He then checks the number and does not put the dice back and repeat the experiment one more time. Let the number appeared at the second dice be N_2. What is the probability that the $N_1 + N_2 \geq 8$.

3. Suppose in a bag there are mathematical elements and numbers as given in Table 3:

Table 3: Mathematical elements in bag

Element	$\times$	-	$+$	2	8	9	1	5
Quantity	2	4	2	1	4	3	7	3

(a) [1 Marks] What is the probability that a person draws 2 operators and 3 numeric elements in first 5 trials [without replacement]?

(b) [1 Marks] How do you apply three random elements chosen from the bag to get a number ≥ 10?

(c) [2 Marks] Suppose a person Sam takes out an element and finds out it to be a mathematical operator '$\times$'. He performs the same operation once again and this time also, he gets mathematical operator '$\times$'. Third time Sam takes out 5 elements together. What is the probability that using the 5 elements Sam will be able to form a number ≥ 7?

4. Five prisoners, Jack, Jill, John, Joy and James, are held in separate cells. Police knows specifically that out of the five, three prisoners are to be executed and two will be freed but prisoners have no idea about who will be executed and who will be free. Each prisoner is brought to trail at a gap of one hour in a random manner. What is the probability that:

(a) [1 Marks] Suppose James and Joy is the prisoner to be freed and any of the five prisoners can come for the trail. What is the probability that in the two prisoners will be executed in sequentially.

(b) [2 Marks] Suppose James and Jill are already given their verdict. Now, Joy is given the information that: The probability for him to be free is 1/3. And at least one of John and Jill will be executed. If he gets the information about one of the prisoner who will be executed from the police, the probability of his execution will be half. Thus, his chance of being free will be high. Investigate Joy's thinking.

(c) [2 Marks] Suppose James and Jill are given their verdict. Now, the policeman forgets that the name of the person to be executed and freed. In trials, prisoners are assigned random verdict. Find the probability that each person receives correct verdict.

(d) [2 Marks] Suppose we now consider 50 prisoners and every prisoner knows the verdict of prisoner presented in the trail before him (For e.g., Fifth prisoner knows the verdict for all the 4 prisoners before him). In each case, if the prisoner will be able to guess correctly about his trial, he will be freed else will be executed. Formulate a strategy for the prisoners such that no prisoner is executed.

5. [2 Marks] For the following Fig. 1, where $X \in [0, 2]$, calculate the CDF of X.

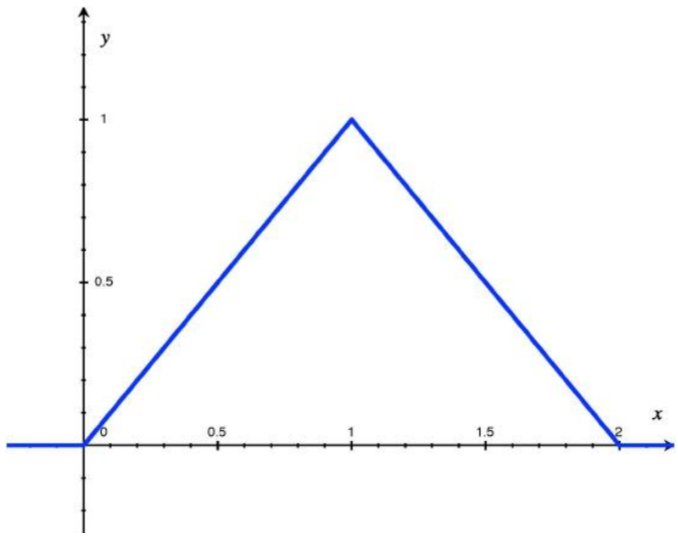

Figure 1: Probability distribution function of X

Programming question

1. Using MATLAB or Python simulate a fair dice experiment with the probability of each output $X = \{1, 2, 3, 4, 5, 6, \text{edge}\}$ as equal.

(a) [2 Marks] Simulate the experiment for 1000 iterations. For each iteration note the output. Plot the histogram for the output.

(b) [2 Marks] Let $Y = \{Y_1, Y_2, Y_3\}$ denote the following events: Y_1 : probability of getting a number 2 on the dice, Y_2 : probability of getting a number 3 on the dice, and Y_3 : probability of getting a number 5 on the dice. Show Y as a 3×1 vector at the output.

(c) [2 Marks] Repeat the experiment for 100 and 10000 iterations and show the histogram for each experiment.

2. [3 Marks] Repeat the experiment for a biased dice with the probability for each outcome as given in Table 4. Plot the histogram for outcomes for number of iterations = 100, 10000.

Table 4: Outcome of unbiased dice

Outcome	1	2	3	4	5	6	edge
Probability	0.3	0.1	0.1	0.2	0.2	0.05	0.05