For each problem show all work. Homework must be submitted in a Word document

1. A binary search of a sorted array with 3 unique elements where the search is always successful can be viewed as an array with each element having a probability of being selected = 1/3 and the *number of comparisons* a binary search requires to find the element is shown inside the array.

| | | |
|---|---|---|
| 2 | 1 | 2 |

Then the *average* number of comparisons would be **A**(3) = (1/3) [2 + 1 + 2] = 5/3

Find the **A**(7) for an array with 7 elements for the same binary search.

(a) Draw the 7-element array showing the *number of comparisons* needed to find each element.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |

(b) Determine **A**(7)

(c) Using what you learned from parts (a) and (b), find the summation expression for **A**(n). To simplify your work, assume that $n = 2^k - 1$. Use k in your summation. Leave your answer in closed form in terms of n.

(d) Simplify your a(n) for large values of n.

2. Consider the algorithm below

```
Precondition: n is a non-negative integer

function f ( n )
{
      temp = 0

      if (n != 0)
      {
         for (i = 1; i <= 3; i++)
            temp = temp + n * f(n-1)

         return temp
      }
      else
         return 1;
```

Solve for the closed form by repeated substitution

3. The recurrence relation is given as

$$a_n = 2a_{n-1} + 3a_{n-2}$$

Use the method linear homogeneous characteristic roots to solve for the closed form, with given initial conditions

$a_0 = 2$ and $a_1 = 4$

(a) Find the general solution

(b) Find the Specific solution

(c) Use your closed form result to find $a_5$

4. Two average complexities are given below. Variable *p* is probability value in the interval of [0.0, 1.0]. Variable *n* is the problem size.

$$A_1 = p(n^2 + 1) + (1-p)3n$$

$$A_2 = (1-p)(3n^2 + n) + p(2n)$$

(a) Determine the condition for which $A_1$ is faster than $A_2$.

(b) Approximate the range of *p* values for which $A_1$ is faster than $A_2$ for large values of *n*.

(c) Given $n=5$, find the range of values for *p* where $A_2$ is faster than $A_1$.

(d) Can you determine the problem size *n* where $A_1$ is always faster? Why or why not?