1. Consider an MA(1) process yt = 10 + ut + 3ut−1, where the values ut are independent and normally distributed N(0; 4).

a) Calculate the theoretical autocorrelation function of the ACF process, ρk.

b) Calculate the first two values of the partial autocorrelation function PACF, ϕ11, ϕ22.

c) Generate a trajectory of this process with a length of 30 observations. Plot the series, plot the first ten values of the sample ACF and PACF.

d) Repeat the previous point for 300 observations. Is it true that as the number of observations increases, the sample ACF converges to the true ACF, and the sample PACF converges to the true PACF?

2. Consider a random walk yt = 1 + yt−1 + ut, where the values ut are independent and normally distributed N(0; 4), and y0 = 10.

a) Calculate E(yt), Var(yt), Cov(y10, y20).

b) Compare Corr(y10, y20) and Corr(y110, y120).

c) Generate a trajectory of this process with a length of 30 observations. Plot the series, plot the first ten values of the sample ACF and PACF.

d) Repeat the previous point for 300 observations. Is it true that as the number of observations increases, the sample ACF converges to the true ACF, and the sample PACF converges to the true PACF?

3. Take any non-seasonal series of annual periodicity. You can take a number from https://fedstat.ru/ or other sources.

a) Plot the series, sample ACF and PACF plots.

b) Visually assess if there is a trend? Does the process look like a stationary one?

c) Estimate the model for the ETS(AAN) series.

d) Write down the resulting equation using the estimated values ​​of the parameters instead of the parameters.

e) Get the 80% confidence interval for one and two steps forward "by hand", based on the written equations.

f) Get the 80% confidence interval one and two steps ahead with built-in functions.

g) Plot the forecast graph and the series itself.

4. Take any seasonal series of quarterly or monthly frequency.

a) Construct the decomposition of the series into components using the STL algorithm. Visualize the result for three different strengths of seasonality adjustment. Briefly comment.

b) Plot the decomposition of the series into components using the ETS(AAA) model.

c) Divide the data into training and test sets, allocating two years of observations to the test set.

d) Estimate the ETS(AAA), ETS(MAM), seasonal naive model and apply Theta method with default STL decomposition and ETS(AAA) for the logarithm of the series.

e) For each approach, find the MASE on the test set.

f) Plot a forecast that averages the forecasts of the two MASE-leading approaches. Did you manage to beat the two averaging approaches?