ECE 2104
Theory of Structures I

Topic 2: Bending Structures

Conditions of Equilibrium

- For a system of coplanar forces this may be expressed by the three equations of static equilibrium

$$\sum H = 0$$

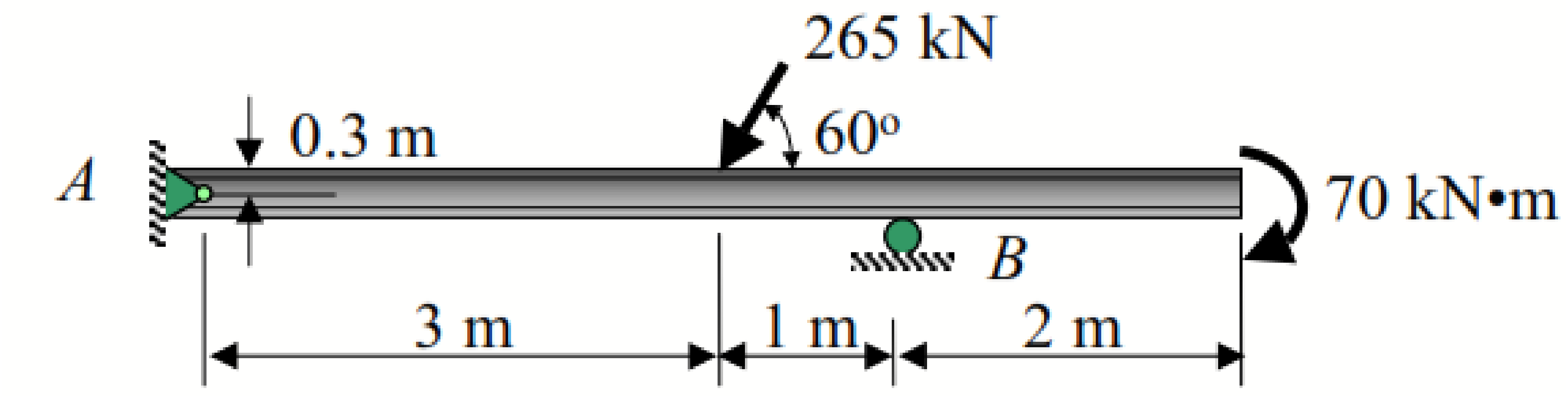
$$\sum V = 0$$

$$\sum M = 0$$

- where
  - H and V are the resolved components in the horizontal and vertical directions of a force and
  - M is the moment of a force about any point

Bending Structures

- **Example 1**
- Determine the reactions of the beam shown



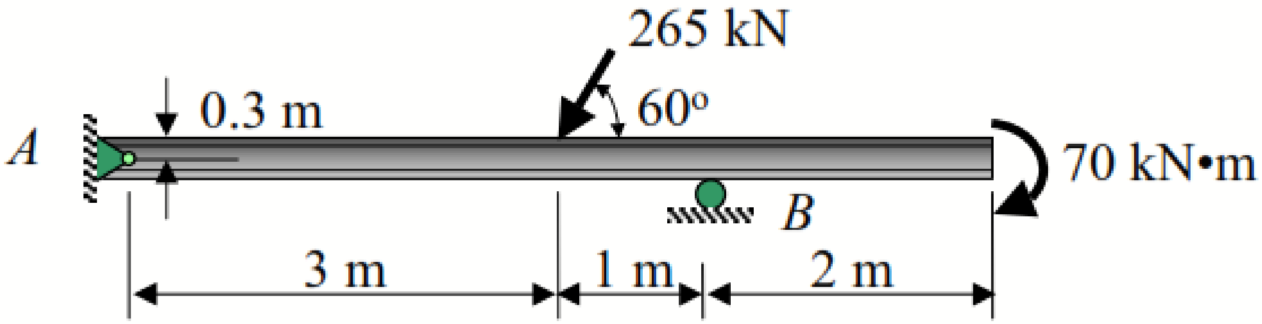
Bending Structures

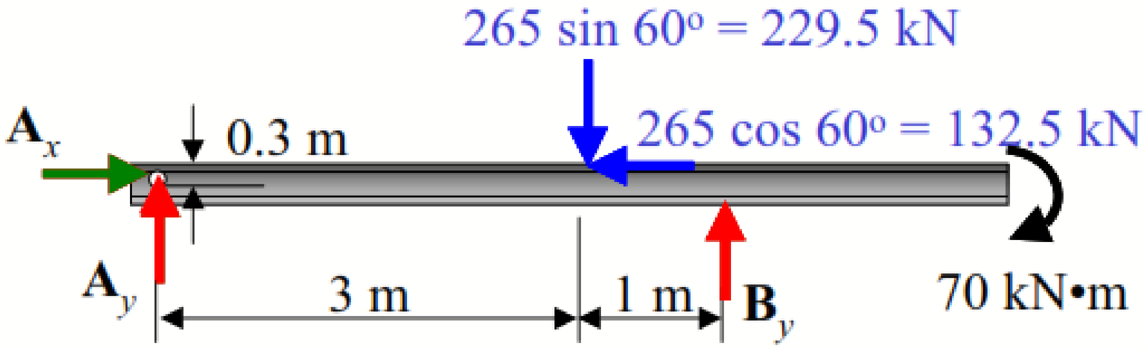
- Equilibrium conditions
- $r = 3n$
- Resolve the loads
- Apply the equilibrium equations

$$\sum H = 0$$

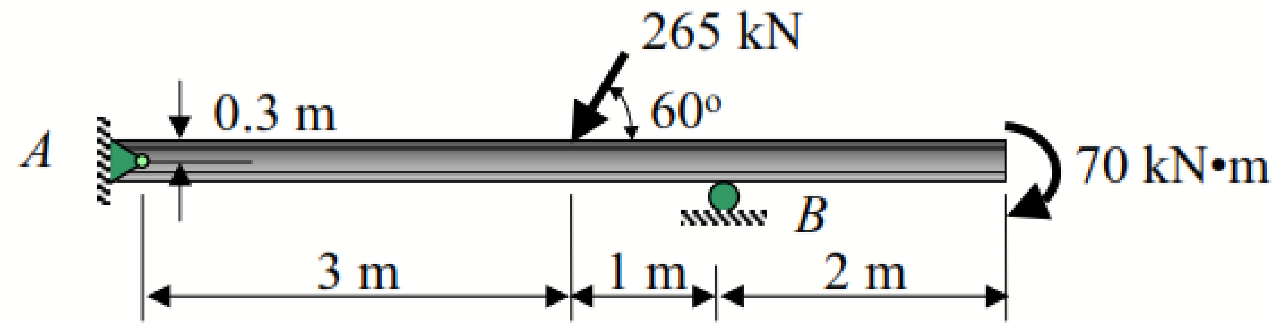
$$\sum V = 0$$

$$\sum M = 0$$

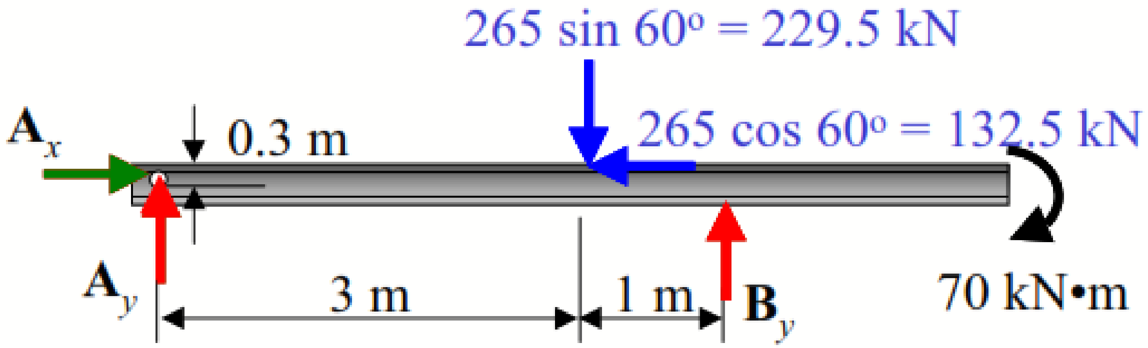




Bending Structures

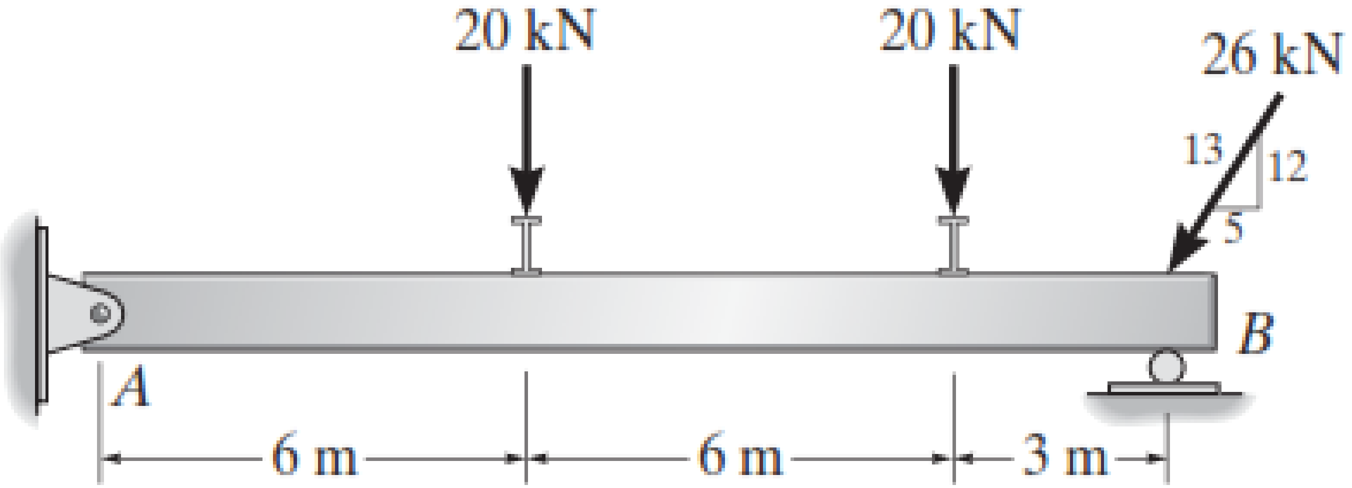


- Apply the equilibrium equations
- $\sum F_x = 0 \ (\rightarrow is + ve)$
- $A_x - 132.5 = 0$;
- $A_x = 132.5\ kN$
- $\sum M_A = 0(\circlearrowleft +ve)$
- $4 \times B_y - 3 \times 229.5 + 132.5 \times 0.3 - 70 = 0$
- $B_y = 179.69\ kN$
- $\sum F_y = 0 \ (\uparrow is + ve)$
- $A_y - 229.5 + B_y = 0$
- $A_y = 49.81\ kN$

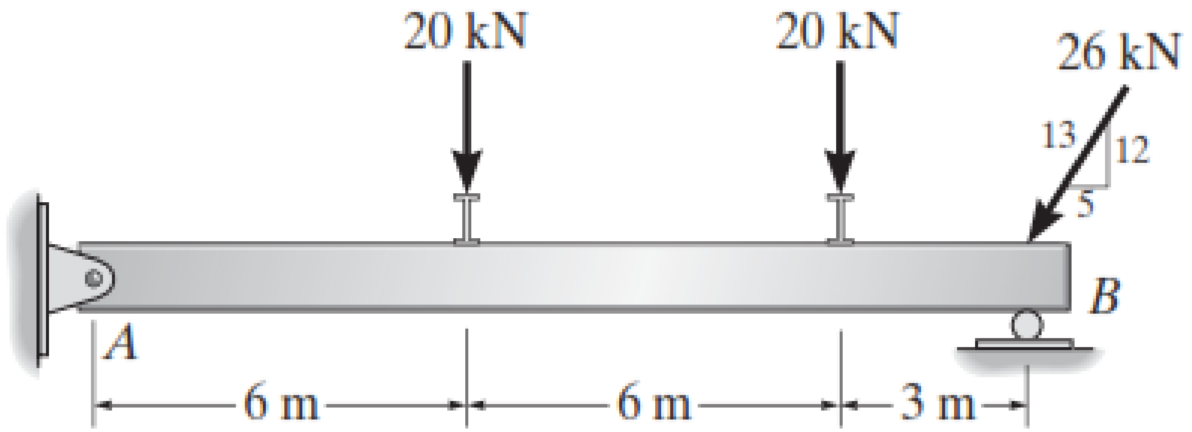


Bending Structures

- **Example 2**
- Determine the reactions of the beam shown. Neglect the thickness of the beam



Bending Structures



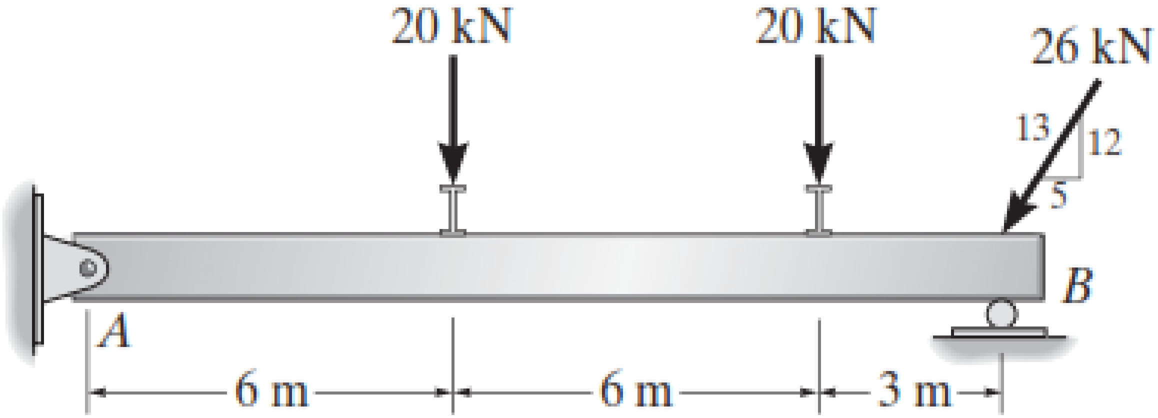
- Equilibrium conditions
- $r = 3n$
- Resolve the load at support B
- $P_v = 26 \times \frac{5}{13} = 10kN$
- $P_h = 26 \times \frac{12}{13} = 24kN$
- Apply the equilibrium equations

$$\sum H = 0$$

$$\sum V = 0$$

$$\sum M = 0$$

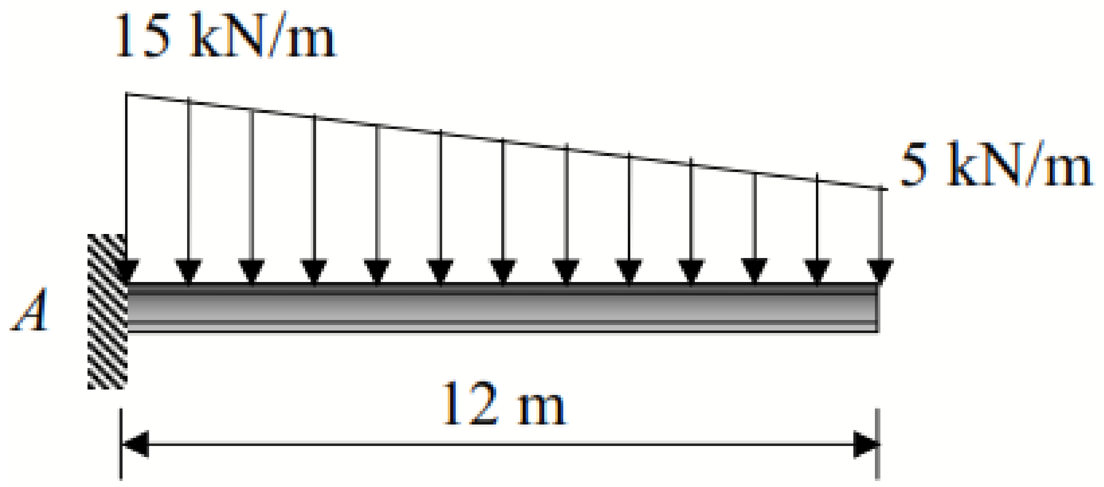
Bending Structures



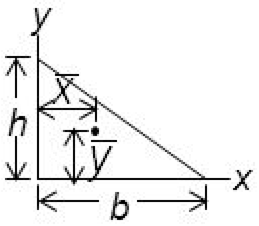
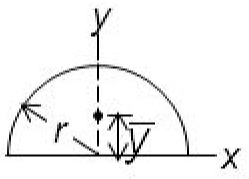
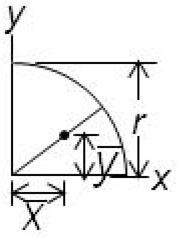
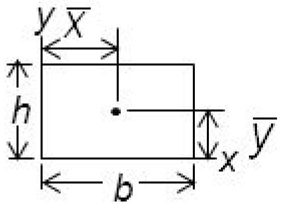
- Apply the equilibrium equations
- $\sum F_x = 0 \ (\rightarrow is + ve)$
- $A_x - 24 = 0\,;$
- $A_x = 24\ kN$
- $\sum M_A = 0(\circlearrowleft +ve)$
- $15 \times B_y - 6 \times 20 - 12 \times 20 - 15 \times 10 = 0$
- $B_y = 34\ kN$
- $\sum F_y = 0 \ (\uparrow is + ve)$
- $A_y - 20 - 20 - 10 + B_y = 0$
- $A_y = 16\ kN$

Bending Structures

- **Example 3**
- Determine the reactions of the beam shown. Neglect the thickness of the beam



Centroid of load

	Shape	$\bar{x}$	$\bar{y}$	Area A
1. Triangle		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2}bh$
2. Semicircle		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
3. Quarter circle		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
4. Rectangle		$\frac{b}{2}$	$\frac{h}{2}$	bh

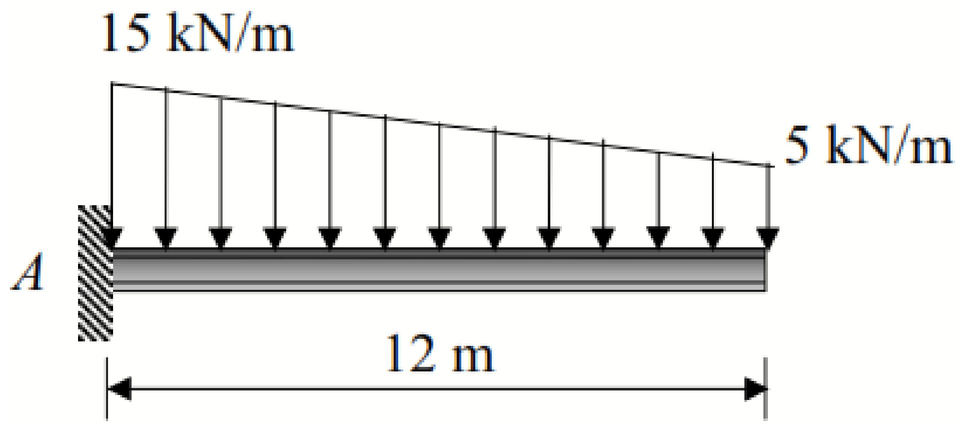
Bending Structures

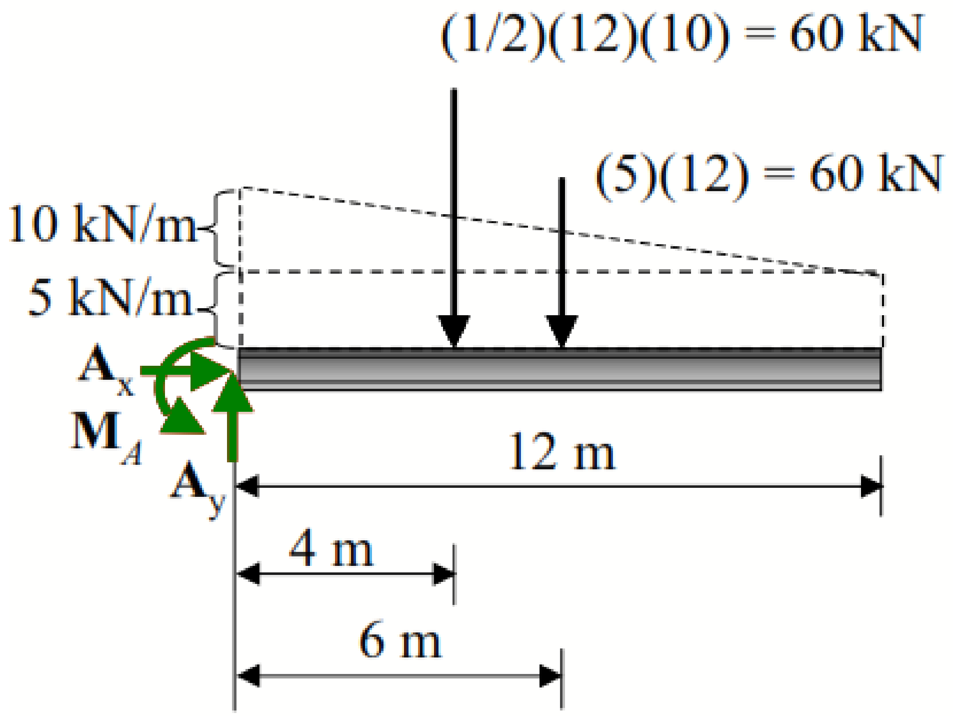
- Equilibrium conditions
- $r = 3n$
- Split the trapezoidal load into rectangle and triangle section
- Determine center of gravity of each load section
- Apply the equilibrium equations

$$\sum H = 0$$

$$\sum V = 0$$

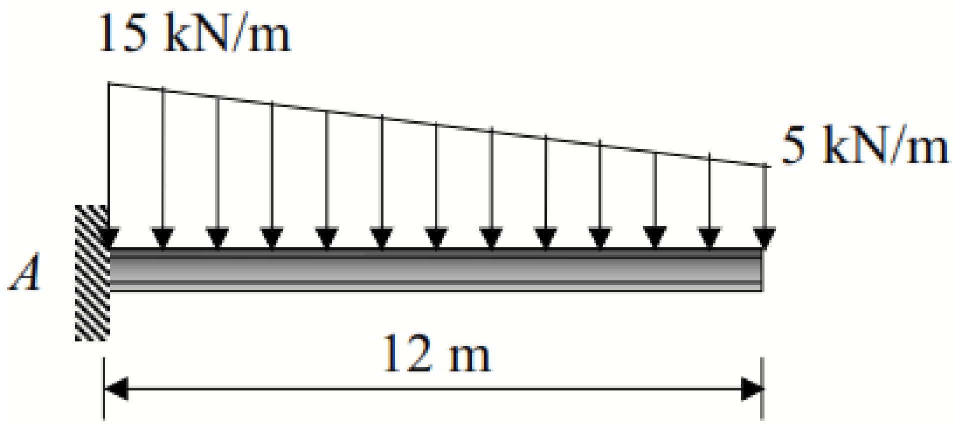
$$\sum M = 0$$

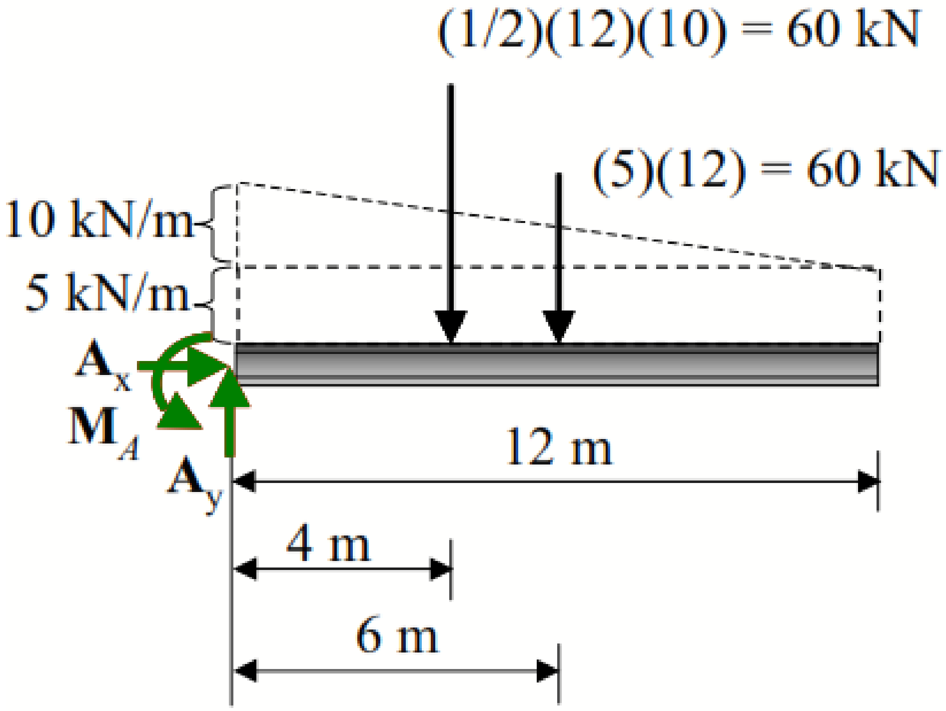




Bending Structures

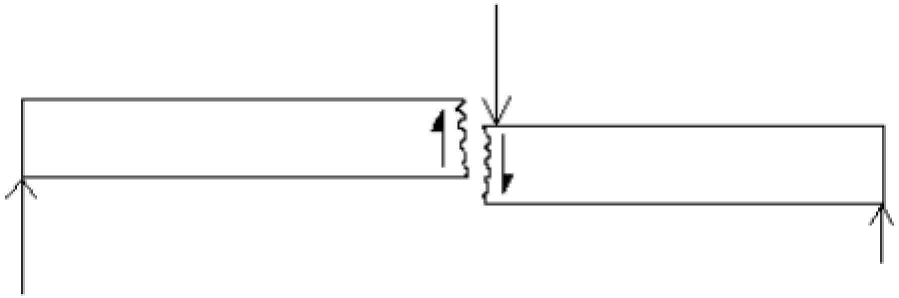
- Apply the equilibrium equations
- $\sum F_x = 0 \ (\rightarrow is + ve)$
- $A_x = 0 \ kN$
- $\sum F_y = 0 \ (\uparrow is + ve)$
- $A_y - \left(\frac{1}{2} \times 12 \times 10\right) - (5 \times 12) = 0$
- $A_y = 120 \ kN$
- $\sum M_A = 0(\circlearrowleft +ve)$
- $M_A - \left(\frac{1}{3} \times 12\right)\left(\frac{1}{2} \times 12 \times 10\right) - \left(\frac{1}{2} \times 12\right)(5 \times 12) = 0$
- $M_A = 600 \ kN.m$

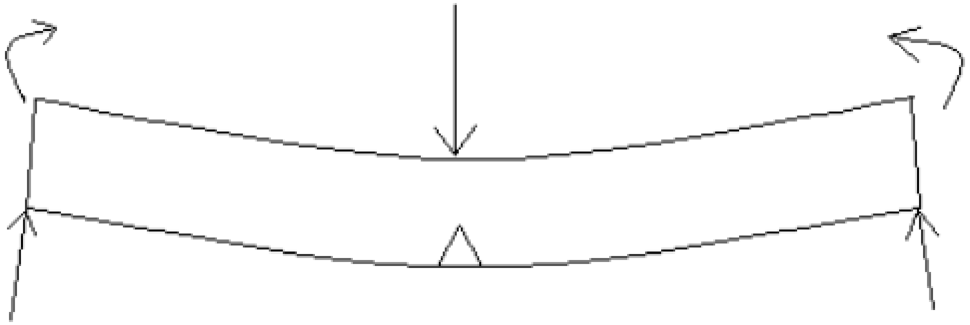




Bending Structures

- Loading tends to cause failure in two main ways
  - by shearing the beam across its cross-section
  - by bending the beam to an excessive amount





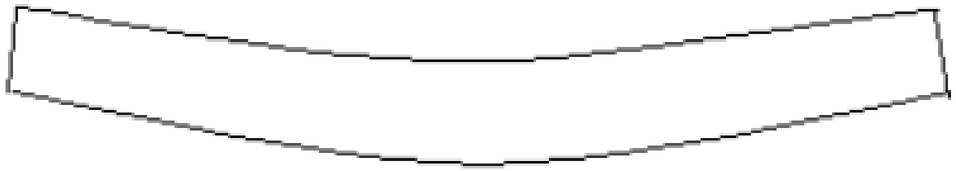
- Shear forces and bending moments are examples of internal forces that are induced in a structure when loads are applied to that structure.

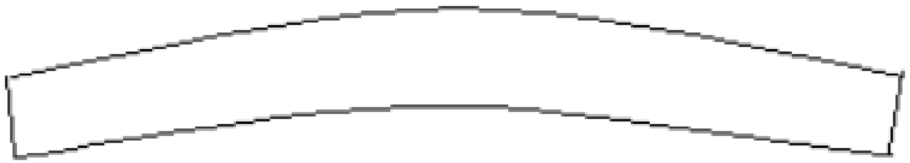
Bending Structures

- **Shear force** may be defined as "the algebraic sum of the loads to the left or right of a point (such that the addition of this force restores vertical equilibrium)“
- A **shear force diagram** is one which shows variation in shear force along the length of the beam.
- **Bending moment** may be defined as "the sum of moments about that section of all external forces acting to one side of that section“
- A **bending moment diagram** is one which shows variation in bending moment along the length of the beam

Bending Structures

- Before shear force and bending moments can be calculated the reactions at the supports must be determined
- The maximum bending moment occurs at the point of zero shear force
- For beams spanning between two simple pin-jointed supports (i.e. no cantilevers) moment will always be positive

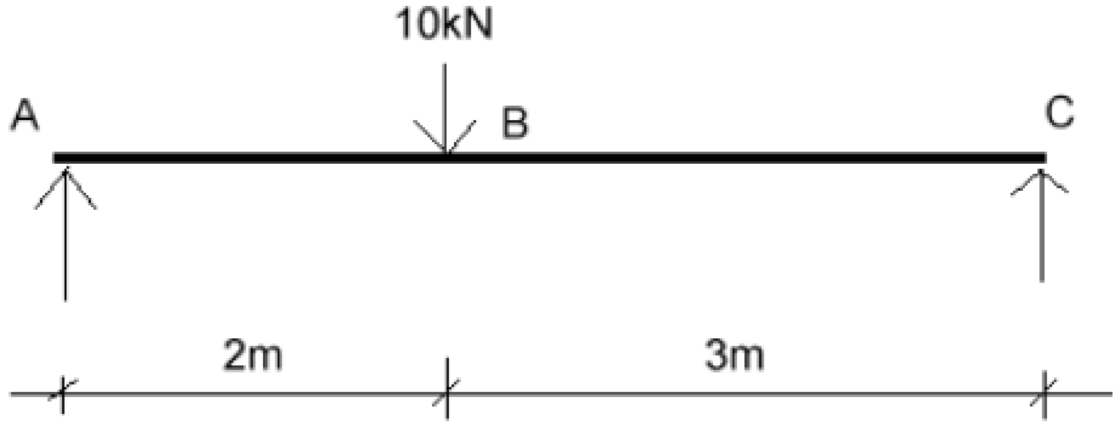




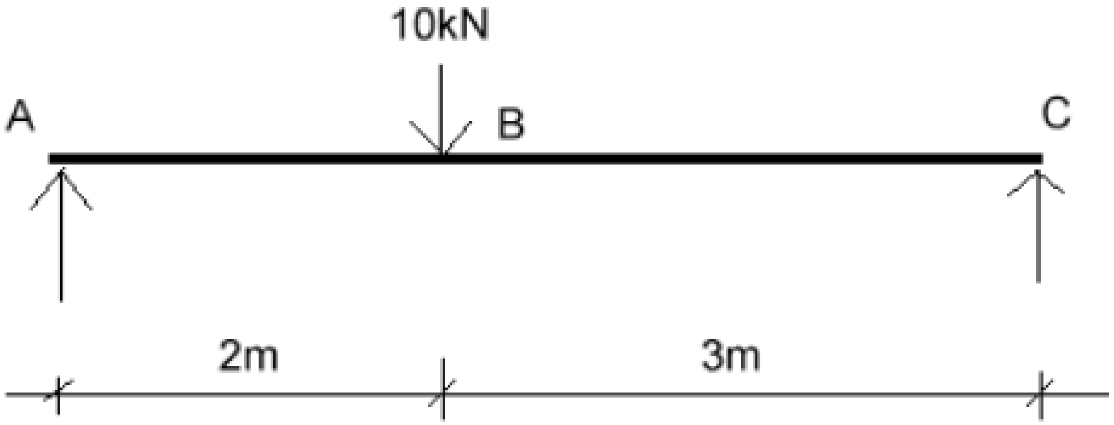
- Sagging (+ve moments) hogging (-ve moments)

Bending Structures

- **Example 4**
- Draw the shear force and bending moment diagrams for the beam shown below



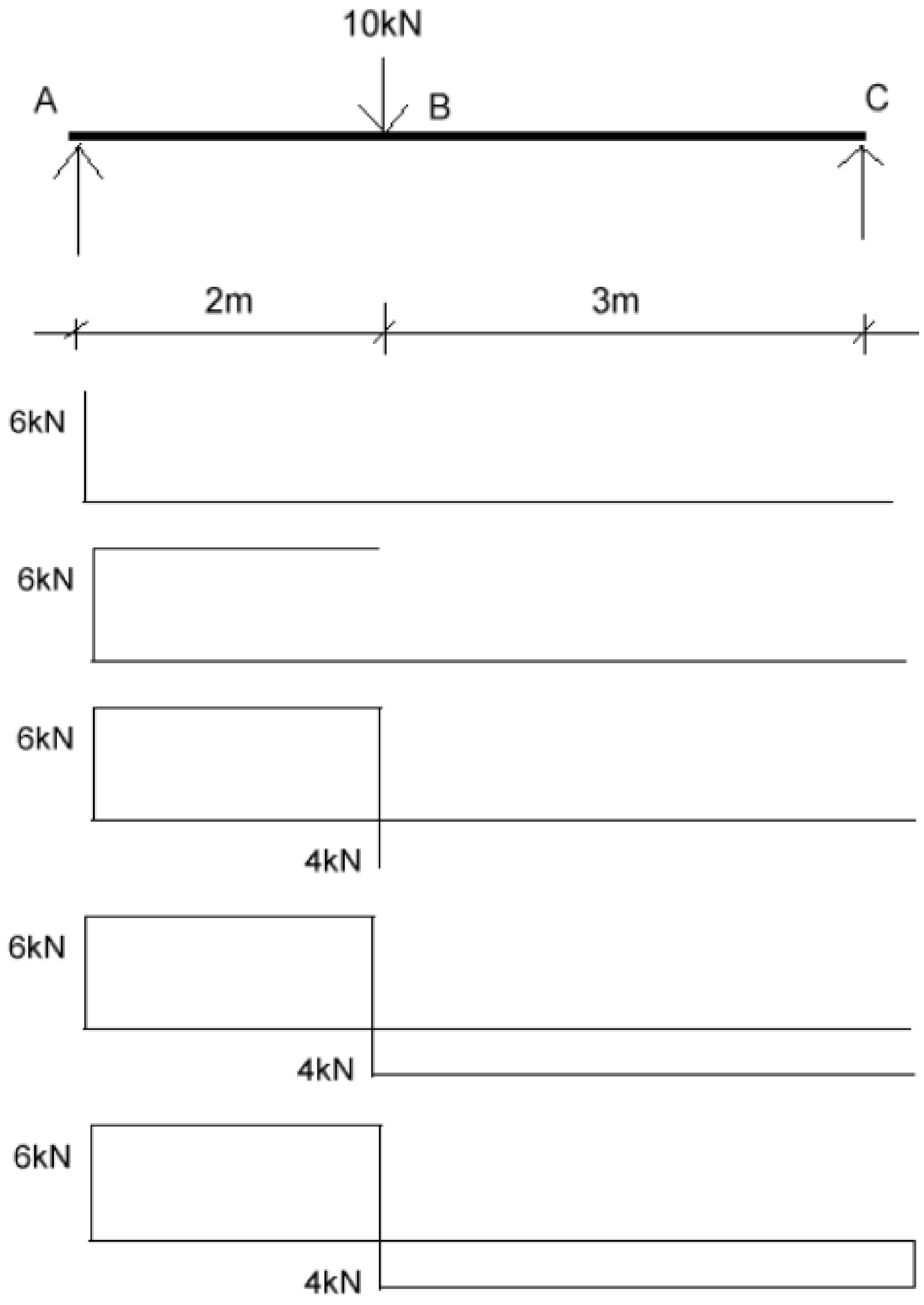
Bending Structures



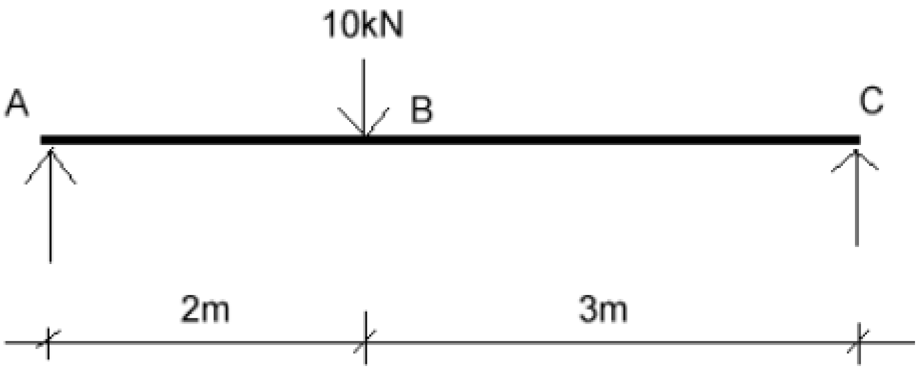
- Apply the equilibrium equations
- $\sum F_x = 0 \ (\rightarrow is + ve)$
- $A_x = 0 \ kN$
- $\sum M_A = 0(\circlearrowleft +ve)$
- $5 \times C_y - 2 \times 10 = 0$
- $C_y = 4 \ kN$
- $\sum F_y = 0 \ (\uparrow is + ve)$
- $A_y - 10 + C_y = 0$
- $A_y = 6 \ kN$

Bending Structures

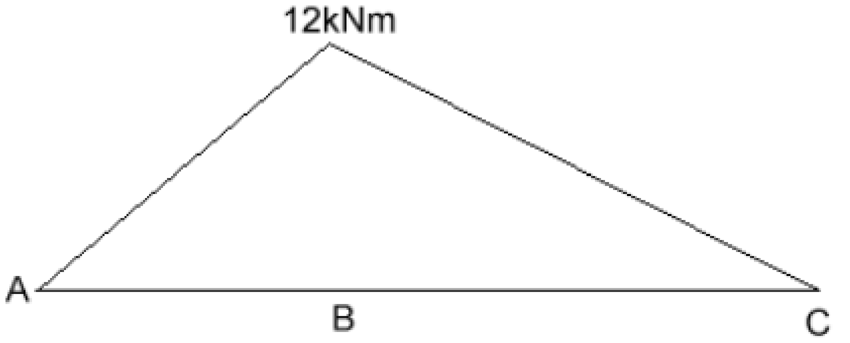
- Draw the shear force diagram, working from left to right
- Reaction at A, $A_y = 6\ kN$
- No force acts between A and B, thus shear force is constant
- At B, 10kN load is applied, thus shear is
- $6 - 10 = -4kN$
- No force acts between B and C, thus shear force is constant
- At C, the reaction C_y acts on the beam, thus shear force is
- $-4 + 4 = 0\ kN$



Bending Structures



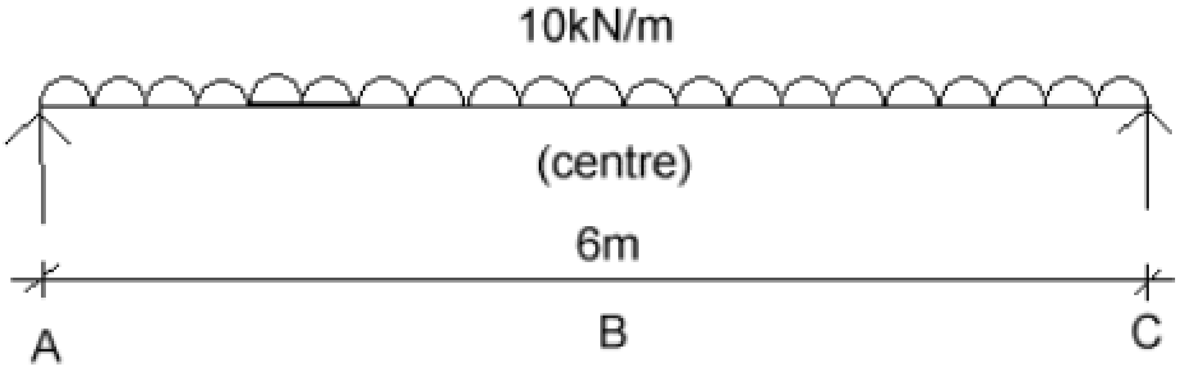
- Draw the bending moment diagram, working from left to right
- Support A is a pinned support, thus $M_A = 0\ kN.m$
- Support C is a pinned support, thus $M_c = 0\ kN.m$
- Determine the bending moment at intervals along the length of the beam



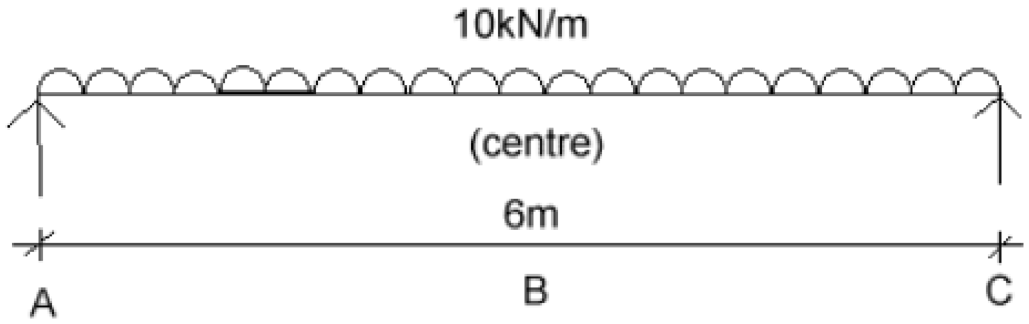
Position	**A**	**A + 1m**	**B**	**B + 1m**	**B + 2m**	**C**
Bending Moment(kN-m)	0	$1A_y = 6$	$2A_y = 12$	$3A_y - 10 = 8$	$4A_y - 2 \times 10 = 4$	$5A_y - 3 \times 10 = 0$

Bending Structures

- **Example 5**
- Draw the shear force and bending moment diagrams for the beam shown below



Bending Structures

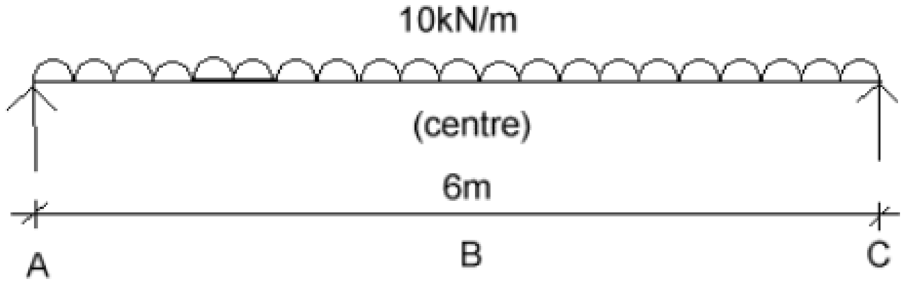


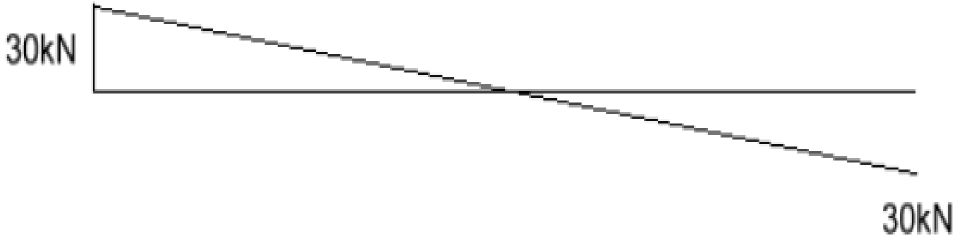
- Apply the equilibrium equations
- $\sum F_x = 0 \ (\rightarrow is + ve)$
- $A_x = 0\ kN$
- $\sum M_A = 0(\circlearrowleft +ve)$
- $6 \times C_y - \left(6 \times 10 \times \frac{1}{2} \times 6\right) = 0$
- $C_y = 30\ kN$
- $\sum F_y = 0 \ (\uparrow is + ve)$
- $A_y - (10 \times 6) + C_y = 0$
- $A_y = 30\ kN$

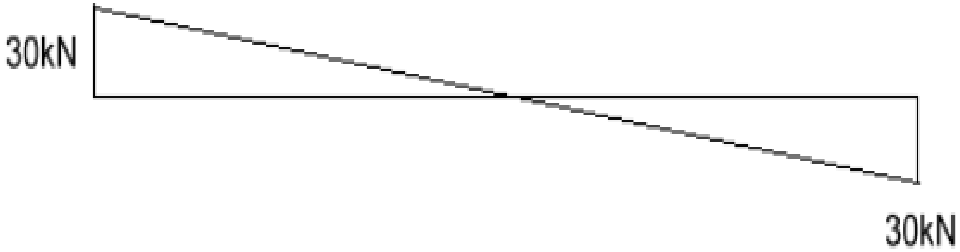
- Note
- For symmetrical loading, reactions are equal

Bending Structures

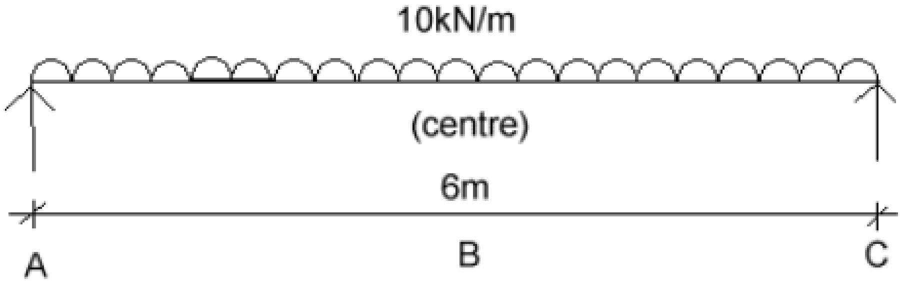
- Draw the shear force diagram, working from left to right
- Reaction at A, $A_y = 30\ kN$
- The applied UDL reduces the shear force by 10kN per metre, thus between A and C
- $30 - (6 \times 10) = -30\ kN$
- At C, the reaction C_y acts on the beam, thus shear force is
- $-30 + 30 = 0\ kN$



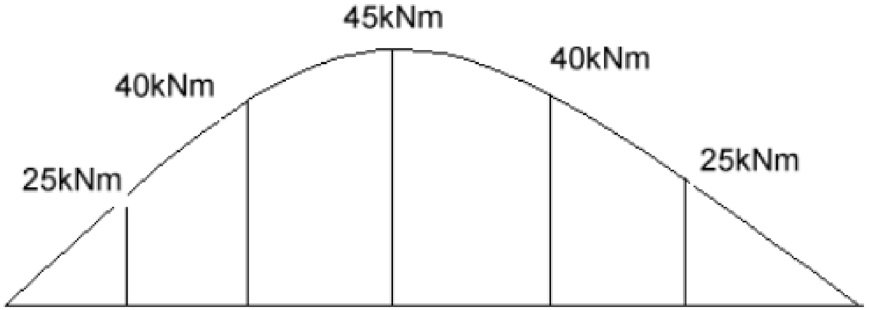




Bending Structures



- Draw the bending moment diagram, working from left to right
- Support A is a pinned support, thus $M_A = 0\ kN.m$
- Support C is a pinned support, thus $M_c = 0\ kN.m$
- Determine the bending moment at intervals along the length of the beam



Position	A	A + 1m	A + 2m	B	B + 1m	B + 2m	C
Bending Moment (kN-m)	0	$1A_y - (10 \times 1 \times 0.5 \times 1) = 25$	$2A_y - (10 \times 2 \times 0.5 \times 2) = 40$	$3A_y - (10 \times 3 \times 0.5 \times 3) = 45$	$4A_y - (10 \times 4 \times 0.5 \times 4) = 40$	$5A_y - (10 \times 5 \times 0.5 \times 5) = 25$	0

- Bending moment does not vary uniformly between A and B and between B and C but the bending moment diagram is parabolic (curved)

Bending Structures

- **Exercise 1**
- Draw the shear force and bending moment diagrams for the beams shown