Lab 3: Surface Energy Balance and Biogeophysical Climate-Vegetation Interactions

Due Date: 5:30pm, 15 Mar 2022

Answer all the questions posted in this exercise and submit all your computer code. Please make sure all plots submitted should have a proper title, axis labels and (for maps) legend with the correct unit or labels. Remember to include your name and student ID.

In class and described in greater detail in Wallace & Hobbs Ch.10.3 and Bonan Ch.27.3, the concept of biogeophysical feedback and the role of life in stabilizing climate can be heuristically illustrated by a hypothetical daisyworld. We will examine this further in this lab exercise.

Let us imagine a planet without an atmosphere, but somehow one type of plant can grow on it, namely, white daisy. The daisy is completely reflective of solar radiation with an intrinsic albedo of one. The surface of the planet is otherwise black, completely absorptive of solar radiation with an intrinsic albedo of zero. Therefore, the planetary albedo (r) is simply equivalent to the fractional coverage of white daisy on the planet's surface. The climate of the planet in terms of surface temperature (T_s, in °C) is governed by the **surface energy balance** equation:

$$\sigma T_s^4 = (1 - r)F_0$$

where F_0 (in W m^{-2}) is the solar insolation (i.e., incoming solar radiative flux) and σ = 5.670×10^{-8} W m^{-2} K^{-4} is the Stefan-Boltzmann constant. The surface energy balance equation is shown as the blue curves in Fig. 1 for different values of F_0.

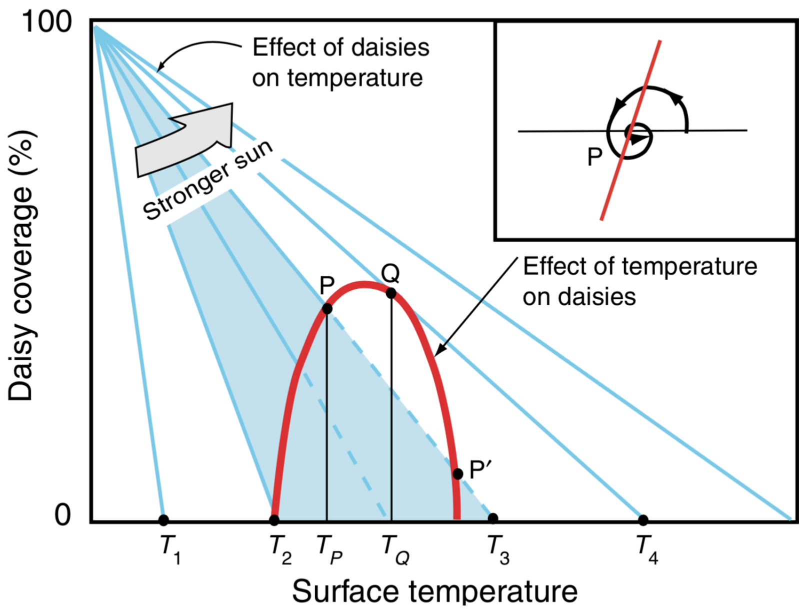

Figure 1. Graphical representation of climate-vegetation equilibria in the daisyworld. The blue curves represent the surface energy balance equation for different values of solar insolation (F_0), and the red curve represents the fractional daisy coverages as a quadratic physiological function of surface temperature. An equilibrium is where the two curves intersect.

The abundance of white daisy is governed by **plant physiology**. Let us assume that fractional daisy coverage (r) takes the form of a quadratic function between a minimum ($T_{\min}$) and maximum ($T_{\max}$) temperature, below and above which coverage would be zero. That is,

for $T_s \leq T_{\min}$: $r = 0$

for $T_{\min} < T_s < T_{\max}$: $r = aT_s^2 + bT_s + c$

for $T_s \geq T_{\max}$: $r = 0$

where a, b and c are quadratic coefficients that are dependent on $T_{\min}$, $T_{\max}$ and the maximum possible fractional coverage, $r_{\max}$. The physiological function is represented by the red curve as well as the parts of the x-axis that are below $T_{\min}$ and above $T_{\max}$ in Fig. 1.

Theoretically, a climate-vegetation equilibrium exists at any point where the surface energy balance curve and physiological curve intersects (e.g., as represented by the points T_1, T_2, P, Q, P', T_3 and T_4). However, numerically (and often in reality), only an equilibrium that is stable can be achieved, because any small perturbation (due to numerical errors in the computer, or climate fluctuations in reality) on an unstable equilibrium would kick off a cascade of feedback that leads the system further away from the unstable equilibrium. A perturbation on an unstable equilibrium can generally lead two possible outcomes. Either the system may simply diverge and then settle into another equilibrium that is stable; or the system may endlessly oscillate between two different states. If the curves are drawn properly, whether an equilibrium is stable or unstable, as well as which of the two outcomes will arise from a perturbation on an unstable equilibrium, can indeed be diagnosed graphically from the shapes of the curves.

In this exercise, we will explore how a gradually increasing solar insolation would affect the climate-vegetation equilibrium in the daisyworld. To save you time, three functions are already provided to you in *Lab03.R*, as explained below. Please make sure you examine the code carefully to understand the algorithms and what they do.

I. Plant physiological function:

plant_physiol(Ts, Ts_min=0, Ts_max=50, alb_max=0.2)

which calculates the fractional daisy coverage (i.e., planetary albedo) as a function of surface temperature T_s (variable *Ts*; in °C), with default parameter values for $T_{\min}$ (*Ts_min*) = 0°C, $T_{\max}$ (*Ts_max*) = 50°C and $r_{\max}$ (*alb_max*) = 0.2.

II. Surface energy balance equation:

energy_balance(albedo, insolation)

which calculates the surface temperature (in °C) as a function of planetary albedo (variable *albedo*) and solar insolation (variable *insolation*, in W m^{-2}).

III. Stable climate-vegetation equilibrium:

equilibrium(F0, alb0, tol=1e-9)

which solves for the climate-vegetation equilibrium, i.e., to find a stable solution (T_s, r) between the plant physiological and surface energy balance equations, given a specific value of solar insolation (variable *F0*, in W m^{-2}) and an initial guess value for planetary albedo (variable *alb0*). This function gives a vector of length two, with the first and second entry being the equilibrium T_s and r, respectively. In the case of oscillatory solutions without a stable equilibrium, the lower of the two temperatures and the corresponding albedo will be given as the output by default, but the other solution will also be printed out. Parameter *tol* is the numerical tolerance allowed for convergence to a solution.

1. Using the function *plant_physiol(...)* and the built-in function *plot(..., type="l", col= "red")*, plot the red curve for the physiological function with surface temperature (in °C) in the x-axis and daisy coverage in the y-axis from $T_s = -10$°C to $T_s = 60$°C.

2. In the **same** plot, using *energy_balance(...)* and build-in function *matplot(..., type="l", col="blue", add=TRUE)* function, plot the blue curves for surface energy balance for different values of solar insolation, from $F_0 = 200$ W m^{-2} to $F_0 = 700$ W m^{-2} in increments of $\Delta F_0 = +20$ W m^{-2}. You should be able to produce a plot that looks somewhat similar to Fig. 1.

3. Now let us simulate what would happen to the climate of daisyworld as F_0 increases from 200 W m^{-2} to 700 W m^{-2}. Using function *equilibrium(...)* and an initial guess value of $r = 0$, first find the equilibrium surface temperature and planetary albedo for $F_0 = 200$ W m^{-2}. Then, for every increment of $\Delta F_0 = +20$ W m^{-2} until $F_0 = 700$ W m^{-2}, find the equilibrium (T_s, r) likewise, always using the equilibrium albedo for the previous F_0 value as the initial guess value for r (if the equilibrium albedo is *NaN* for the previous F_0 value, using $r = 0$ as the initial guess).

4. Based on your results from part 3 above, plot the equilibrium temperature (in the y-axis) against different values of solar insolation (in the x-axis). Also draw in the **same** plot a dotted line for the hypothetic curve of equilibrium temperature vs. solar insolation if no white daisy exists at all (i.e., $r = 0$ always). Describe and explain physically how the very existence of white daisies affects the climate of this planet. Moreover, describe the relationship between the equilibrium temperatures you found here and the various intersection points in the figure you drew for part 2.