Exam 1

Assigned: Oct. 22th, 2021

Due: Oct. 31, 2021 at 11:59PM

Submission instructions: Refer to webcourses.

Note: The point values for each problem maybe slightly adjusted.

1. Consider the following LP [12pts]

$$
\begin{aligned}
\max \quad & 5x_1 + 8x_2 + 6x_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 && \leq 10 \\
& -x_1 + 2x_2 + 4x_3 && \leq 10 \\
& 4x_1 + 4x_2 + 2x_3 && \leq 8 \\
& x_1, x_2, x_3 && \geq 0
\end{aligned}
$$

   (a) Solve the LP using the simplex method

       i. What is the optimal solution?

       ii. What is the optimal objective value?

       iii. What is the solution to the corresponding slack variables?

       iv. What are the shadow prices? Indicate where you found the values in the simplex tableau by boxing/circling the values.

   (b) What are the optimal basic variables?

2. Consider the following LP [11pts]

$$\begin{array}{ll}
\max & 5x_1 + 8x_2 + 6x_3 \\
\text{subject to} & x_1 + x_2 + x_3 \leq 10 \\
& -x_1 + 2x_2 + 4x_3 \leq 10 \\
& 4x_1 + 4x_2 + 2x_3 \leq 8 \\
& x_1, x_2, x_3 \geq 0
\end{array}$$

(a) Program and solve the LP in max normal form using CPLEX. For your solution:

   i. Print your code as PDF

   ii. Include a screenshot off the solution

(b) Program and solve the LP in standard form using CPLEX. For your solution:

   i. Print your code as PDF

   ii. Include a screenshot off the solution

   iii. What is the solution to the corresponding slack variables?

(c) Using the CPLEX Interactive Optimizer (using the max normal LP formulation), provide screenshots of the following:

   i. The optimal solution (all variables at same time, *hint*: display solution variables -)

   ii. The optimal dual variable solution (all variables at same time)

   iii. The optimal slack variable solution (all variables at same time)

   iv. Results from sensitivity analysis of the objective function (all variables at same time)

   v. Results from sensitivity analysis of the right-hand side (all constraints at same time)

3. Consider the following LP [19pts]

$$
\begin{aligned}
\max \quad & 5x_1 + 8x_2 + 6x_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 && \leq 10 \\
& -x_1 + 2x_2 + 4x_3 && \leq 10 \\
& 4x_1 + 4x_2 + 2x_3 && \leq 8 \\
& x_1, x_2, x_3 && \geq 0
\end{aligned}
$$

Answer the following questions "manually" without using CPLEX, while using your knowledge of the optimal basis from problem 1.

(a) Using the Dual Theorem, what is the optimal dual solution?

(b) By how much can the objective coefficient for x_1 change while keeping the same optimal basis (in other words, what is Δ)?

(c) By how much can the objective coefficient for x_2 change before the optimal basis (in other words, what is Δ)?

(d) By how much can the rhs of the 2nd constraint change while keeping the same optimal basis (in other words, what is the allowable Δ)?

(e) Assume we add an activity corresponding to the decision variable x_4, with objective coefficient $c_4 = 10$, and constraint coefficients 2, 4, and 1 corresponding to the 1st, 2nd, and 3rd constraints. Will there be a change to the optimal basis?

4. Bob the carpenter makes chairs and stools. During any month he can make a **total** of 65 pieces of furniture (chairs and stools). The material costs to make the furniture and demand for the furniture varies each month according to the tables below. It costs $5 to store a chair and $4 to store a stool in inventory each month. [8pts]

Month	chairs	stools
1	30	25
2	34	28
3	38	34

Table 1: Monthly Cost

Month	chairs	stools
1	40	20
2	30	30
3	20	10

Table 2: Monthly Demand

(a) Formulate an LP to minimize the total cost of satisfying the demand for the 3 months.

(b) Program and solve the LP in CPLEX

5. Consider the following infeasible LP [Extra Credit]

$$
\begin{array}{lll}
\text{max} & 5x_1 + 8x_2 + 6x_3 & \\
\text{subject to} & x_1 + x_2 + x_3 & \geq 10 \\
& -x_1 + 2x_2 + 4x_3 & = 10 \\
& 4x_1 + 4x_2 + 2x_3 & \leq 8 \\
& x_1, x_2, x_3 & \geq 0
\end{array}
$$

(a) Attempt to solve using the Two-Phase Simplex method. (This is a harder problem. Might have to read book a few times)

(b) From attempting the Two-Phase Simplex method how can you tell the problem is infeasible?