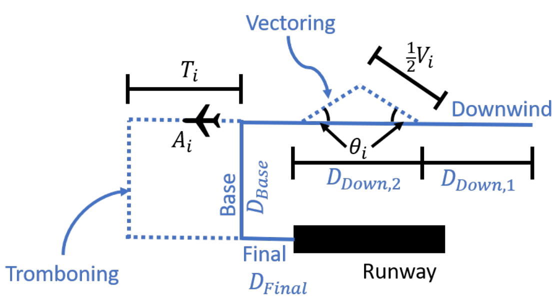
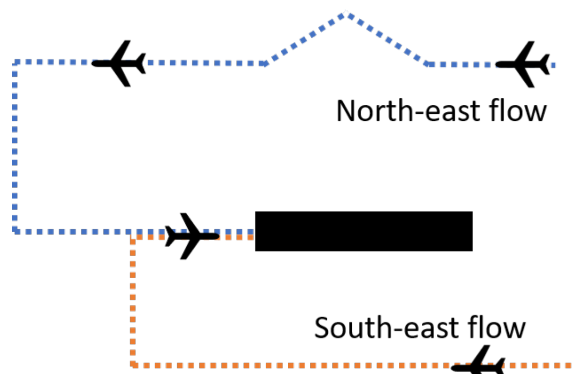
Extended Assignment
Applications of Mixed Integer Linear Programming
Assigned: Nov. 11th, 2021
Due: Dec. 10, 2021

Submission instructions: Refer to webcourses.

Here we will consider the case of scheduling the aircraft arrivals at Vela Regional Airport (VRA), a large airport with a dedicated arrivals runway. A standard air traffic landing pattern for the arrival runway is illustrated below in Figure 1a. The pattern consists of three basic segments: (1) the downwind; (2) the base; and (3) the final approach. Air traffic controllers at the airport are able to temporally space aircraft at the runway (to prevent collisions) through a combination of *vectoring* and *tromboning*. Vecotoring is a path-lengthening technique that extends the time required to traverse the downwind segment. Similarly, tromboning extends the path length of both the downwind segment and the final approach segment.



(a) A standard arrival pattern for VRA



(b) A standard arrival pattern for VRA

At VRA there are two primary arrival flows coming from the North-East and South-East of the airport as shown in Figure 1b. In order to ensure safety, aircraft must be spaced at the runway. The separation required between pairs of aircraft depends on the wake category of each aircraft, and which aircraft is the leader and follower. For example, when a large aircraft is followed by a small aircraft, the small airplane must spaced further away when landing in order to make sure the turbulent wake from the large aircraft does not cause the small aircraft to crash. Meanwhile, if the small aircraft lands first, less distance between the aircraft is required because the large aircraft is not as affected by the smaller turbulent wake generated by the small aircraft. An illustration of this concept is provided in Figure 2. Note, the separation between the aircraft is not defined in time, but rather in units of distance (Nautical Miles, NM).

In the next 24 hours N_1 and N_2 aircraft (denoted A_i) are scheduled to land at VRA using the North-east flow or South-east. Each aircraft A_i arrives at the entry to their arrival flow at time t_i while traveling at a speed of s_i; the aircraft hold their speed constant until they land. At a minimum, aircraft A_i will travel a total distance of $D_{Down,1} + D_{Down,2} + D_{Base} + 2D_{Final}$ (assume the nominal length of both flows is equal). As shown in Figure 1a, the downwind segment

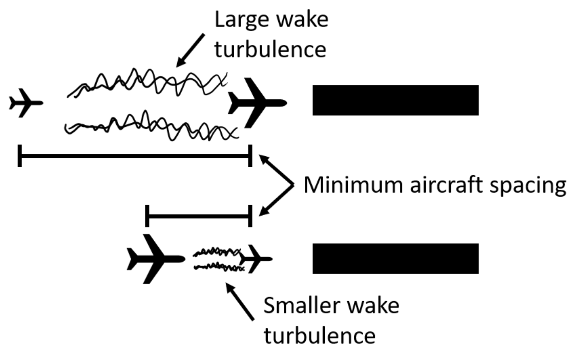


Figure 2: The minimum required separation between aircraft depends on the size of the aircraft, the the size of the turbulent wake generated by the leader aircraft.

is divided into two parts, one which is fixed length ($D_{Down,1}$), and another which is variable length ($V_i \geq D_{Down,2}$) where vectoring can occur. In order to space aircraft at the runway, the total distance an aircraft travels may be extended through the use of vectoring and tromboning. So for an aircraft A_i, the extra distance traveled is given by $V_i - D_{down,2} + 2T_i$ where V_i is the length of the vectoring segment, and T_i is the distance the final approach segment is extended.

The goal of this project is to write a Mixed Integer Linear Program (MILP) that minimizes fuel-burn of arriving aircraft, while ensuring they are properly separated at the arrival runway.

For this problem you will be provided the following values in a CPLEX dat file (these should be referenced in your written formulation):

- N_1: the number of aircraft arriving to VRA on the north-east flow
- N_2: the number of aircraft arriving to VRA on the south-east flow
- t_i [Hr of day]: the arrival time of A_i to it's cooresponding flow.
- s_i [NM/Hr]: the groundspeed of A_i.
- $D_{Down,1}, D_{Down,2}, D_{Base}, D_{Final}$ [NM]: fixed-value distances related to each arrival flows.
- $D_(i,j)^s$ [NM]: the minimum separation distance required between two arriving aircraft, where A_i the leader and A_j the follower, at the point in time when aircraft A_i lands.
- f_i [kg/NM]: the fuel burned per unit distance traveled.

The MILP must enforce the following constraints:

- Aircraft operations are safely separated in units of distance at the runway.
- Aircraft on the same flows cannot pass each other.
- The heading angle change, θ, when vectoring is restricted to 30^o.
- The maximum distance the final approach segment can be extended is 15NM

As part of your optimization formulation, for each aircraft A_i, you should solve for:

- The time the aircraft lands [Hr]
- The total distance the aircraft travels [NM]
- The extra distance traveled along the final approach segment [NM]
- The extra distance traveled when vectoring along the downwind segment [NM]

Please note, even through the problem is described mostly in units of distance [NM], your primary decision variables can be defined in units of time [Hr].