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A Mixed Integer Program for Flight-Level Assignment and Speed Control for Conflict Resolution

Adan Vela, Senay Solak, William Singhose, John-Paul Clarke

Abstract— **We consider the air traffic conflict resolution problem and develop an optimization model for generating speed trajectories that minimize the fuel expended to avoid conflicts. The problem is formulated by metering aircraft at potential conflict points. The developed model is a mixed integer linear program that can be solved in near real-time for large number of aircraft.**

I. INTRODUCTION

Development of advanced air traffic conflict detection and resolution algorithms is imperative to overall health, management, and improvement of the air traffic management (ATM) system, both in Europe and the United States. Eurocontrol and the Federal Aviation Administration (FAA), the respective governing bodies for air traffic control, have recognized the importance of providing some level of automation. In particular, computer aided conflict resolution stands out as a key component of next generation systems, i.e. the Single European Sky ATM Research (SESAR) in Europe [1], and the Next Generation Air Transportation System (NextGen) in the United States [2]. The need for advanced algorithms is especially relevant when one considers issues of safety and capacity within the context of growing air traffic. From 2006 to 2020, the number of instrument flight rules (IFR) aircraft operations handled by FAA enroute traffic centers is predicted to increase from approximately 46 million operations to 70 million operations [3]. As such, the increased demand on the National Airspace System (NAS) will require air traffic controllers to maintain greater situational awareness and to provide safe and robust conflict resolution in real-time. In Europe, similar discussion of enroute capacities leading to enroute delays is reported in [4]. Excluding weather, many of the enroute delays are attributed to staff shortages and lack of capacity planning. It is expected that automated conflict resolution systems will aid in reducing delays while allowing air traffic controllers handle larger workloads.

There exist numerous research and proof-of-concept papers detailing various air conflict resolution models. A comprehensive survey of many proposed models is presented in [5]. Since the publication of that survey, numerous other methods have been introduced and discussed. Some of these methods include tactical approaches focusing on resolving immediate conflicts, with limited consideration of future post-conflict routing and planning [6]. In [7], the authors make use of genetic algorithm to solve a complex multi-aircraft problem with a series of altitude and heading changes. In [8], a linear programming approach is proposed for route and flight level assignments modeled as a flow problem with a space-time network to minimize costs of congestion, potential conflict, and routing. While the developed model is effective in long-term planning and flight-level assignment, it does not specifically address how potential conflicts may be resolved. Two speed-control methods for conflict resolution include [9] and [10]. In [9], the authors consider utilizing speed control in a method similar to our proposed approach, however, it does not proceed beyond a linear model. Linear programming constraints force aircraft to preserve any original ordering at intersection points and only considers a single flight level. In [10], the authors focus on resolving local pairwise conflicts under uncertainty, but do not consider multiple aircraft. Finally, in another related approach, [11], the authors make use of scheduling algorithms to resolve planar aircraft conflicts at intersection points along fixed aircraft paths.

ERASMUS is a related Eurocontrol funded project to study methods for including automation concepts into air traffic control consistent with human factors issues. One of the primary results of ERASMUS is the concept of 'subliminal control'. In this approach significant portions of traffic are deconflicted with minor automated speed control commands ($\pm 6\%$). The speed-change commands of this magnitude are not perceptible to pilots or controllers. Use of minor speed changes to space aircraft can reduce the number of aircraft controllers actively monitor. There exist numerous publications related to the ERASMUS project and subliminal control [10], [12], [13].

Our proposed approach utilizes concepts based on speed control and flight-level assignments for conflict resolution over predefined routes. The developed mathematical model is formulated as a mixed integer linear program (MILP). The MILP model provides a broad framework for resolving conflicts through fast numerical optimization methods. The model allows for adjusting the amount of control provided: assigning arrival times, speed commands for aircraft inside and outside the airspace, and can be applied by iterative or centralized methods. We also focus on medium-term conflict resolution, over the time typically required for an aircraft to traverse an air traffic control sector, and any additional time in which would be beneficial for planing ahead for future

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aircraft, between 15-45 minutes. While there exist several sources of uncertainty in such long-term planning, this model serves as a basis for future stochastic models that are robust. Or conversely, this research highlights the possibility of how strict adherence to speed trajectories is able to resolve conflicts. Particular emphasis is placed on reducing fuel costs and emissions involved in conflict resolution. This is deemed important considering the role that fuel plays in the operating costs of aircraft and growing concern regarding the environmental impact of aircraft emissions.

It should be noted that the conflict resolution problem is highly dimensional. However, by confining aircraft along predefined paths, the size and complexity of the problem is greatly reduced. While rule based methods dominate other solution methods in regard to solution times, advances in computing have provided the ability to solve optimal conflict resolution problems in near real-time [14]. However, our model attempts to take advantage of increased computing power currently available to allow for near real-time solutions even to difficult problems that consider large groups of aircraft.

The remainder of the paper deals with the formulation and implementation of the mathematical model into a form that is solvable by MILP solvers. The formulation takes advantage of the properties of the objective function, and works towards reducing the problem size by minimizing the number of integer and binary variables. It is then demonstrated through case studies that the optimization program can solved efficiently even as air traffic demand increases.

The advancement in this paper is the formulation of a complete optimization model for speed control and flight-level assignment to reduce fuel burn over time horizons between 15-45 minutes.

In Section II, we provide a general description of the problem. In Section III, we describe the mathematical model by detailing separation constraints to ensure safety. Next, in Section IV linear approximations of the objective functions are provided. The model is completed in Section V with descriptions of additional bounds and constraints on variables. Section VI provides a complete summary of the problem formulation. In Section VII an analysis of the algorithm and computational studies are presented, and in Section VIII, our conclusions.

II. Problem Description

We consider the air conflict resolution problem, and develop an optimization model to minimize expected fuel costs by issuing required time of arrivals (RTA), flight level assignments, and speed commands. Within the considered airspace, aircraft flight-levels will be static over their trajectory.

Consider a set of aircraft, $\mathcal{A}$, as shown in Figure 1, located outside, inside, or at the border of the airspace, designated by $\mathcal{W}$, with corresponding predefined flight paths. The set of aircraft within the interior airspace is designated by $\mathcal{A}^{in}$, aircraft at the boundary by $\mathcal{A}^{b}$, and aircraft outside the airspace by $\mathcal{A}^{o}$. The primary task is to assign each aircraft a speed profile that ensures conflict-free travel, while minimizing a measure of fuel-burn costs over all the trajectories. We propose a time-based conflict resolution model to meter aircraft at intersection points. As shown in Figure 1, the intersection points between aircraft trajectories define potential conflict regions. For simplicity, we place restrictions on intersecting aircraft trajectories: aircraft intersect only once at a single crossing point; or aircraft fly the same trajectory, e.g. aircraft flying in trail along the same route. The relaxation of this restriction is still solvable with a simple augmentation of the problem to include multiple crossing points. The problem size will increase slightly by the number of additional unique intersections or trailing sections. For aircraft that trail each other over a fraction of their trajectories, the proposed conflict constraints can be expressed as a combination of trailing and intersecting constraints.

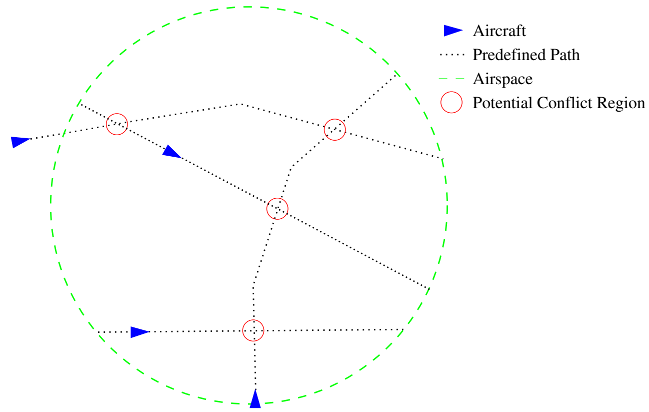

Fig. 1: All aircraft will be assigned speed commands, while aircraft outside will also be issued RTA's and Flight Level assignments

The trajectory of an aircraft i is deemed to be conflict-free if the distance between aircraft i and any other aircraft j maintains a minimum separation greater than D^s over the trajectory. The minimum separation distance is defined by the controller. For the purpose of commercial enroute travel the minimum separation distance between aircraft is 5 NM. Given the problem description, the separation requirement for two aircraft i and j at the same flight level can be stated as follows:

$$\sqrt{x_{dist}(t)^2 + y_{dist}(t)^2} \geq D^s \quad \forall t \in \mathcal{T}^{enroute} \tag{1}$$

where $x_{dist}(t)$ and $y_{dist}(t)$ represents the distance between the two aircraft in the corresponding coordinate axes:

$$\begin{aligned} x_{dist}(t) &= x_i(t) - x_j(t) \\ x_{dist}(t) &= y_i(t) - y_j(t) \end{aligned} \tag{2}$$

Given predefined trajectories, and constraints on aircraft speed, it is possible to calculate the separation time required between aircraft intersecting at a single common point in space in order to ensure conflict free flight. For speeds v_i and v_j, intersection angle $\theta_{i,j}$, and linearly extrapolated

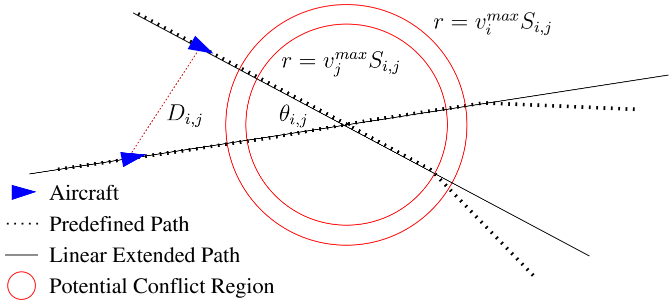

Fig. 2: Aircraft trajectories are assumed to be linear within potential conflict areas

trajectories, the minimum separation time between aircraft is given in [11] as:

$$\hat{S}_{i,j} = \frac{D^s}{v_i v_j \left|sin(\theta_{i,j})\right|} \sqrt{v_i^2 + v_j^2 - 2 v_i v_j cos(\theta_{i,j})} \quad (3)$$

An upper bound, $S_{i,j}$, on $\hat{S}_{i,j}$ can be established by varying parameters v_i and v_j based on aircraft operating bounds, so $S_{i,j} = max\left\{\hat{S}_{i,j} : v_i \in V_i, v_j \in V_j\right\}$.

We assume that the trajectories of the aircraft are linear within a neighborhood conflict region. If the separation time between aircraft is, $S_{i,j}$, then the trajectory of aircraft i cannot have any turning point within a distance of $v_i^{max} S_{i,j}$ about the intersection point as shown in Figure 2.

Each aircraft i is defined and associated with the following variables and functions:

1) Feasible arrival times: $T_{i,0} \in \left(T_{i,0}^{min}, T_{i,0}^{max}\right)$
2) Feasible airspeeds: $v_i \in \left(v_i^{min}, v_i^{max}\right)$
3) Set of feasible flight level assignments: $\mathcal{L}^i$
4) Convex objective function for the cost of arrival times at flight level l: $G_{i,l}(T_{i,0})$
5) Convex objective functions for fuel cost over trajectory at flight level l: $F_{i,l}(v_i)$
6) Predefined trajectories for each aircraft defined by a sequence of waypoints projected onto the X-Y plane: $P_i = (P_{i,1}, \ldots, P_{i,m}, \ldots, P_{i,M})$, where $P_{i,m} \in \mathcal{R}^2$
7) Minimum separation times at intersection nodes between any pairs of aircraft i and j: $S_{i,j}$.

For future clarity, let us introduce the unordered set $\mathcal{I}^x$ corresponding to the set of non-trailing aircraft with potential conflicts at a single crossing point:

$$\begin{aligned} \mathcal{I}^x = \{(i,j) | i,j \in \mathcal{A}, i \neq j, \exists \{m\}, \{n\} \text{ of cardinality} > 1 \\ \text{s.t. } P_{i,m} = P_{j,n}, P_i \neq P_j, (t_{i,m}^{min} \leq t_{j,n}^{max}) \,\&\, (t_{i,m}^{max} \geq t_{j,n}^{min})\} \end{aligned} \quad (4)$$

Additionally, lets define the unordered set of trailing aircraft with potential conflicts:

$$\begin{aligned} \mathcal{I}^t = \{(i,j) | i,j \in \mathcal{A}, i \neq j, P_i = P_j \\ (t_{i,m}^{min} \leq t_{j,n}^{max}) \,\&\, (t_{i,m}^{max} \geq t_{j,n}^{min})\} \end{aligned} \quad (5)$$

The values $t_{i,k}^{min}$ and $t_{i,k}^{max}$ correspond to bounds on the closed set of feasible metering times for aircraft i to arrive at waypoint k. In other words, (i,j) is in $\mathcal{I}^x$, if it is possible for aircraft i and j to reach a common waypoint at the same time.

Thus, the resulting decision variables from the developed optimization model are:

1) Required arrival times into the airspace for each aircraft: $T_{i,0}$
2) Airspeed commands for aircraft i over each link m: $v_{i,m}$
3) Flight level assignment for aircraft i given by an indicator vector: $L_i = [L_{i,1}, \ldots, L_{i,L}, L_{i,L+1}]$ where $L_{i,l} \in \{0,1\}$ and $\sum_{l=1}^{L+1} L_{i,l} = 1$. The case of $L_{i,L+1}$ will indicate a non feasible solution that may require greater intervention from controllers.

III. Flight Level Assignment and Conflict Constraints

The model must be put into a form that is efficiently solved by mixed integer solvers. The formulation takes advantage of the properties of the objective function, and works towards reducing the problem by minimizing the number of integer and binary variables. The resulting formulation can be implemented using Big-M notation, a description of which can be found in [15]. Binary variables are used to assign flight levels, to indicate if pairs of aircraft fly at different flight levels, and also for deconfliction of aircraft traversing along the same flight level. The conflict resolution problem between pairs of aircraft, (i,j), at a common waypoint will be segmented into three possibilities: i and j operate at different flight levels; i arrives at the waypoint before j; and j arrives at the waypoint before i.

To formally define the conflict and flight level assignment, the variable, $L_{i,j}^d$, acts as a binary indicator variable expressing if aircraft i and aircraft j are assigned to different flight levels. $L_{i,j}^d = 1$ occurs when the condition $L_i \cdot L_j = 0$ is satisfied. The test condition can equivalently be rewritten to be consistent with mixed integer programming methods by the following:

$$\begin{aligned} L_{i,l} + L_{j,l} \leq 2 - L_{i,j}^d \\ \text{or} \\ L_{i,l} - L_{j,l} \geq L_{i,j}^d \end{aligned} \quad (6)$$

which respectively imply that

$$\begin{aligned} L_{i,l} + L_{j,l} \leq 1 \iff L_{i,j}^d = 1 \\ \text{or} \\ L_{i,l} - L_{j,l} \geq 0 \iff L_{i,j}^d = 0 \end{aligned} \quad (7)$$

If the trajectories of aircraft i and aircraft j intersect, i.e. $(i,j) \in \mathcal{I}^x$, and are on the same flight level, $L_{i,j}^d = 0$, then one of the conflict constraints must hold:

$$T_{i,m} + S_{i,j} \leq T_{j,n} \quad \text{or} \quad T_{j,n} + S_{i,j} \leq T_{i,m} \quad (8)$$

where aircraft i and aircraft j intersect at a common node, indicated by the m^{th} and n^{th} node of their corresponding paths, respectively. And the value $T_{i,m}$ is the time for aircraft i to reach the m^{th} way-point on its trajectory. The conflict constraints make use of the separation time required between aircraft, as determined with equation (3). For trajectories based on piece-wise linear segments, the time to way-point, $T_{i,m}$, can be defined as:

$$T_{i,m} = \sum_{k=0}^{m} t_{i,k} \quad \forall i \text{ s.t. } \exists j : (i,j) \in \mathcal{I}^x \tag{9}$$

where $t_{i,k}$ is the time it takes for aircraft i to travel from the $(k-1)^{th}$ waypoint to the k^{th} waypoint in its trajectory if a conflict might occur.

Again, the constraints in (8) require an additional binary variable to handle the two possible options: either aircraft i passing the intersection point first, or aircraft j passing the intersection point first. Clearly these options are exclusive from each other. The binary variable indicating the order is defined by $B_{i,j}$ and $A_{i,j}$, such that $B_{i,j} = 1$ if aircraft i arrives at the intersection point before aircraft j, and $A_{i,j} = 1$ if aircraft i arrives after aircraft j. Note the order constraints complete the three possible conflict resolution options. The conflict constraints are then:

$$\begin{aligned} & L^d_{i,j} + B_{i,j} + A_{i,j} = 1 \\ & \textbf{If } B^c_{i,j} = 1 \\ & \qquad T_{i,m} + S_{i,j} \le T_{j,n} \\ & \textbf{Elseif } A^c_{i,j} = 1 \\ & \qquad T_{j,n} + S_{i,j} \le T_{i,m} \end{aligned} \tag{10}$$

Additional consideration must be payed to trailing aircraft, $(i,j) \in \mathcal{I}^t$. It is intuitive that if aircraft i arrives along the route before aircraft j, then at the very least $T_{i,k} \le T_{j,k}$ along each waypoint in the route. Considering the minimum separation, the constraint at the k^{th} waypoint then becomes:

$$T_{i,k} \le T_{j,k} - \frac{D^s}{D_{j,k}} t_{j,k} \quad \text{or} \quad T_{j,k} \le T_{i,k} - \frac{D^s}{D_{i,k}} t_{i,k} \tag{11}$$

This equation still holds even when $D^s \ge D_{i,k}$.

IV. Objective Function

The objective function considers fuel-burn minimization as a primary objective as part of the broader problem of conflict resolution. Ideally, the proposed framework provides latitude in defining classes of potential cost functions that may take into account system time, fuel-burn, controller workload, or any other metric or combination of metrics. The fuel-burn equations used as part of the optimization formulation are derived from the Base of Aircraft Database (BADA) [16], a database of aircraft performance measures published by EUROCONTROL. The fuel cost equations for cruising aircraft are convex functions of the airspeed, and also groundspeed, v^{gs}, when taking into account wind, v^{wind}, where $(v^{gs} + v^{wind}) \cdot v^{gs} \ge 0$. The general form of the fuel burn equations are given in $[kg/s]$ by:

$$F^{fuel}_i(v_i) = \sum_{k=-2}^{4} c_{i,k}(v_i - v^{wind})^k \quad \forall i \in \mathcal{A} \tag{12}$$

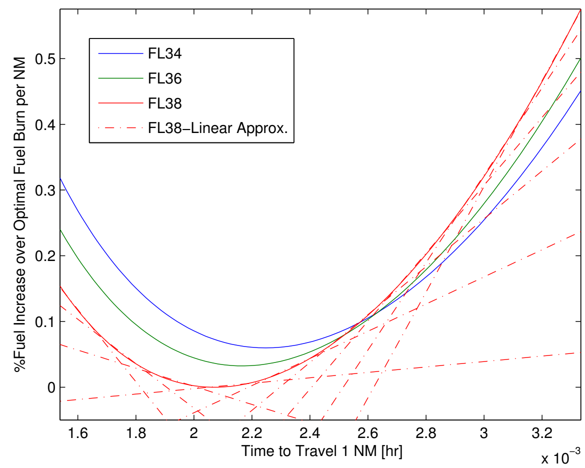

Fig. 3: Sample fuel curves for various aircraft at FL330

where $c_{i,k}$ are constants calculated from various performance and physical parameters of each aircraft.

For cost calculations due to airspeed and flight-level changes, we assume that the cost for each aircraft is the percent deviation in fuel burn per unit distance traveled when compared to the optimal speed and flight-level of the aircraft. $F^{\%f/NM}_{i,l}(v_i) = (F^{fuel/NM}_{i,l}(v_i) + \delta_{FL_l})/F^{fuel/NM}_{i,l^*}(v^*_i) - 1$. The term δ_{FL_i} is any additional cost associated with changing flight levels to FL_l. The original fuel burn functions, $F^{fuel}_{i,l}$, are in units of kg of fuel consumed per minute as a function of the true airspeed. A tranformation can be applied to convert this to Kg of fuel consumed per NM traveled by dividing by the ground speed.

$$F^{fuel/NM}_{i,l}(v_i) = F^{fuel}_{i,l}(v_i)/v_i[Kg/NM] \quad \forall i \in \mathcal{A} \tag{13}$$

The function, $F^{\%f/NM}_i(v_i)$, which is convex in v_i, is also convex in the time variable $t_d = d_i/v_i$. This is true for any class of functions, $F(v)$, that satisfy $-2\frac{d}{dv}F(v) \le v\frac{d^2}{dv^2}F(v)$, which holds true for the class of fuel burn equations we are working with.

We assume the objective function for the complete model with N aircraft is given by $\frac{1}{N}\sum_{i \in \mathcal{A}}(F_i + G_i)$, where F_i is the fuel burn of aircraft i and G_i are any costs related to the required arrival time. The fuel burn for any aircraft must be defined for each of the potential flight levels it can be assigned. Thus, the total percent increase in fuel burn for a single aircraft at level l is $F_{i,l} = \sum_{k=1}^{N}(d_{i,k}/D_i)\, F^{\%f/NM}_{i,l}(d_{i,k}/t_{i,k})$. The function $F^{\%f/NM}_{i,l}(d_{i,k}/t_{i,k})$ can be represented by a set of q linear inequalities defined by slopes $m_{i,l}$ and $b_{i,l}$ based on fuel curves such as the ones shown in Figure 3, where aircraft i corresponds to a particular type, and l to the flight level. For computational and practical purposes, an additional phantom flight level is also included so that $L_i = [L_1, \dots L_L, L_{L+1}]$ where $L_{L+1} = 1$ corresponding to an infeasible solution, i.e. the inability to deconflict aircraft i within reasonable cost limits F_i^{max}. This results in the constraints and cost equations below:

$$
\begin{array}{ll}
F_{i,l} = \sum_{k=1}^{N} \left(d_{i,k}/D_i\right) F_{i,l}^{\%f/NM} & \\
F_{i,l}^{\%f/NM} \geq m_{i,l} \frac{t_{i,k}}{d_{i,k}} + b_{i,k} \quad \forall i,l,k & \\
L_{i,l} \Longrightarrow F_i = F_{i,l}, F_i \leq F_i^{max} & \\
L_{i,L+1} \Longrightarrow F_i = K * F_i^{max} &
\end{array} \tag{14}
$$

V. Additional Constraints and Bounds

The final set of constraints refers to aircraft performance limitations, particularly speed constraints and required time of arrival (RTA) constraints. Given that the MILP model is defined in time, it is necessary to convert $v_i^{min} \leq v_i \leq v_i^{max}$ to be linear in t_i, with the equivalent inequalities:

$$
\frac{1}{v_i^{max}} \leq \frac{t_{i,k}}{d_{i,k}} \leq \frac{1}{v_i^{min}} \tag{15}
$$

It is also possible to include other speed constraints. As in [12], it is possible to enforce only "subliminal" commands, such that all speed changes are within $\pm 6\%$ or some $\triangle\%$, $v_i(1-\triangle) \leq v_{i+1} \leq v_i(1+\triangle)$. Or, constant speed constraints can be enforced, $v_{i,1} = v_{i,k} \forall k$. Using similar transformation into time, the constraints can be expressed to be consistent with the MILP formulation.

As part of the formulation, RTA constraints can also be enforced for aircraft outside the airspace. Based on traffic considerations in adjacent airspaces, distance to considered airspace, and aircraft operating bounds, its is possible to define linear constraints for each aircraft based on the original problem definition as:

$$
T_{i,0}^{min} \leq T_{i,0} \leq T_{i,0}^{max} \tag{16}
$$

VI. Model Summary

The described optimization model can be implemented by formulating the problem through the objective function and constraints described in Sections III, IV and V as shown below:

$$
\begin{array}{l}
min \quad \sum_{i \in \mathcal{A}} C_i F_i + \sum_{i \in \mathcal{A}^c} D_i G_i \\
s.t. \\
\left.\begin{array}{l}
L_{i,j}^d + B_{i,j} + A_{i,j} = 1 \\
\textbf{If } B_{i,j}^c = 1 \\
\qquad T_{i,m} + S_{i,j} \leq T_{j,n} \\
\textbf{Elseif } A_{i,j}^c = 1 \\
\qquad T_{j,n} + S_{i,j} \leq T_{i,m} \\
T_{i,m} = \sum_{k=0}^{m} t_{i,k}
\end{array}\right\} \forall (i,j) \in \mathcal{I}^x \\
\left.\begin{array}{l}
L_{i,j}^d + B_{i,j} + A_{i,j} = 1 \\
\textbf{If } B_{i,j}^c = 1 \\
\qquad T_{i,k} \leq T_{j,k} - \frac{D^s}{D_{j,k}} t_{j,k} \\
\textbf{Elseif } A_{i,j}^c = 1 \\
\qquad T_{j,k} \leq T_{i,k} - \frac{D^s}{D_{i,k}} t_{i,k} \\
T_{i,m} = \sum_{k=0}^{m} t_{i,k}
\end{array}\right\} \forall (i,j) \in \mathcal{I}^T \\
\left.\begin{array}{l}
\textbf{If} \quad L_{i,l} = 1 \quad \forall l \neq L+1 \\
\qquad \left\{\begin{array}{l}
F_i = F_{i,l} \\
F_i \leq F_i^{max} \\
F_{i,l} = \sum_{k=1}^{N} d_{i,k}/D_i F_{i,l}^{\%f/NM} \\
F_{i,l}^{\%f/NM} \geq m_{i,l} \frac{t_{i,k}}{d_{i,k}} + b_{i,k}
\end{array}\right. \\
\textbf{Else} \\
\qquad F_i = K * F_i^{max} \\
\frac{1}{v_i^{max}} \leq \frac{t_{i,k}}{d_{i,k}} \leq \frac{1}{v_i^{min}} \quad \forall k \\
T_{i,0}^{min} \leq T_{i,0} \leq T_{i,0}^{max} \\
\sum_{l=1}^{L+1} L_{i,l} = 1 \\
L_{i,l} \in \{0,1\} \\
O_{i,j} \in \{0,1\}
\end{array}\right\} \forall i \in \mathcal{A}
\end{array} \tag{17}
$$

The above problem is a mixed integer linear program that can be solved with any MILP solver. The **if/elseif/else** conditions are a result of the three options dictated in (10): aircraft fly on difference flight-levels, and the ordering of aircraft arrivals at a metering point.

VII. Computational Results

The performance of the proposed model has been tested on a series of scenarios to simulate realistic air traffic conditions. An air traffic model of ZME19, an enroute sector within Memphis Center, was generated to approximate traffic through the sector. A statistical distribution of the entry-exit pairs was generated using historical data of westbound aircraft traveling through the center at and above FL300 during a 24hr period. The day considered represents a nominal day in the NAS with no significant disruptions. The center boundary is broken into 10NM segments, which are numerically identified as entrances and exits. For the distribution, each aircraft is designated to enter and exit through a particular boundary segment.

Aircraft interarrival times into the sector are assumed to follow an exponential distribution. To increase traffic loads,

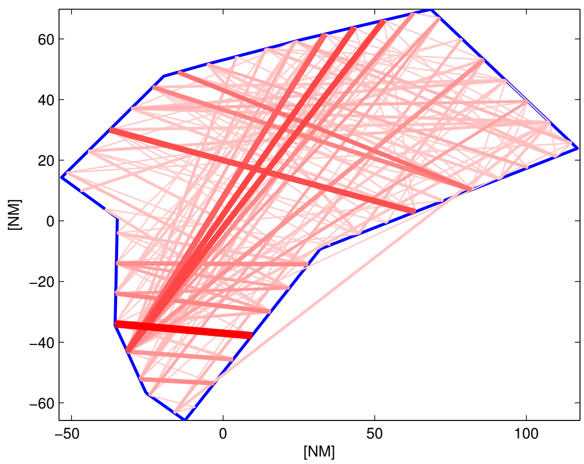

Fig. 4: The sampled distribution of aircraft entry-exit pairs used for simulating a realistic air traffic pattern

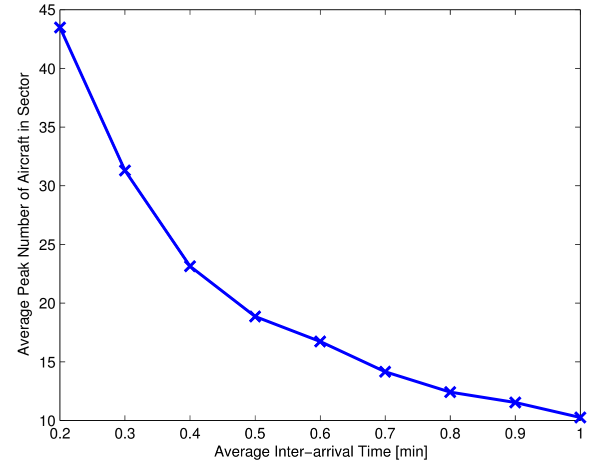

Fig. 5: Average peak number of aircraft in the sector at any given time

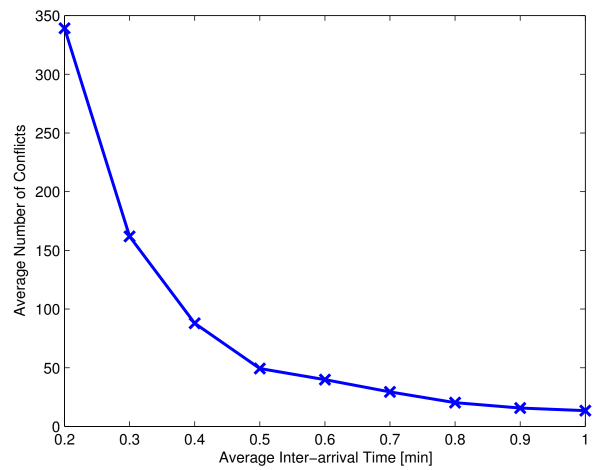

Fig. 6: Average number of conflicts during 30min time period under no control

the average interarrival time between aircraft is decreased. Aircraft models are also assigned according to the sampled distribution taken from the historical data. The aircraft span a large range of models, including: regional, narrow body, wide body, and business class jets. For the purpose of simulations, the aircraft are only sampled from the distribution of aircraft flying westbound. The sampled traffic pattern through ZME19 is displayed in Figure 4. Heavily utilized entry-exit pairs, which yield direct routings, are shown in darker color, while lightly colored lines represent entry-exit routes that are less commonly flown by aircraft. The peak number of aircraft in the sector at any time for the simulated arrival rates is shown in Fig. 5. The traffic levels for the simulations are well beyond the average number of aircraft a controller typically handles in a sector, even during peak times. Due to the large amount of traffic, it is understandable that the number of pairs of conflicts, if no control action is taken, is extremely high, as shown in Fig. 6. This number is generated assuming aircraft maintain the same speed and heading throughout its trajectory despite previous conflicts.

We consider a centralized version of the model with various numbers of flight-levels, and compare the results from these models. The three model options include: 1 flight-level (FL39), 2 flight-levels (FL37, FL39), and 3 flight-levels (FL35, FL37, FL39). Each of the models include the infeasibility flight-level option, $L_{i,L+1}$ with $F_i^{max} = 0.5$ and $K = 100N$, which is described in Section IV. Hence, any aircraft i that cannot be resolved with $F_i \leq 0.5$ will be identified by the model so that it can be extracted and manually resolved by an air traffic controller. The controller's solution may include a more complex resolution based on vectoring, speed changes, and/or flight-level changes at a point in the trajectory. Flights are also restricted to speeds changes of $\pm 40kts$. The initial time of entrance into the airspace, $T_{i,0}$, is assumed to be fixed for each aircraft, removing a level of control. Implied in the removal of RTAs is deletion of G_i from the cost equation. Future discussion is required into the feasibility of assigning arrival windows, $\left(T_{i,0}^{min}, T_{i,0}^{max}\right)$, to aircraft, and the ability of aircraft to satisfy the RTA within a bounded time window of reasonable accuracy. Note that the size of the arrival window is clearly linked with the flexibility in the trajectory of the aircraft prior to entering the considered airspace. By removing the window, we only make the problem more difficult to solve. Hence the inclusion of an entrance window can only improve the solution. The model also does not consider the fuel consumption associated with changing flight levels. As noted in the formulation, this addition is simple to include. The exclusion of the costs associated with flight level changes does not affect the overall complexity of the problem. We consider the following statistical results for each flight-level configuration and arrival rate:

1) Number of infeasible aircraft

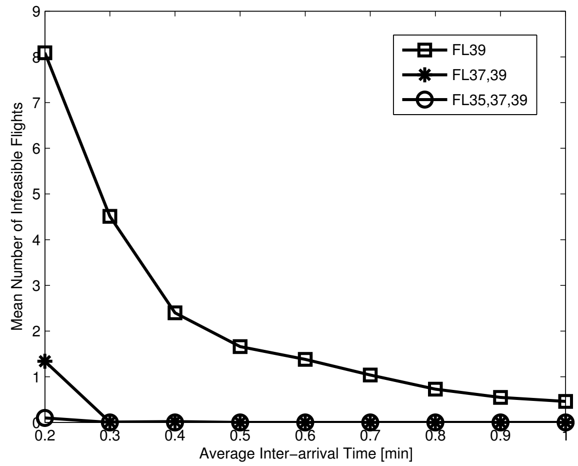

Fig. 7: Average number of infeasible flights versus average inter-arrival time

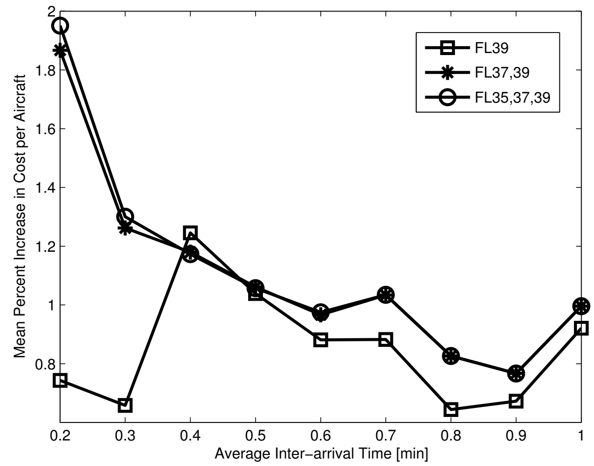

Fig. 8: Average percent increase in Cost per feasible aircraft

2) Average cost per plane, excluding infeasible aircraft

For each flight-level model, 50 traffic cases, with average inter-arrival times of $[0.2, 0.3, 0.4, \dots, 1]$ minutes between aircraft over a 30 minute period are simulated. The corresponding arrival rates of $[300, 200, 150, \dots, 60]$ [aircraft/hour] exceed current traffic levels at a single flight level for ZME19. For some cases, even when divided over 3 flight-levels, the traffic level is still above historic peak traffic levels. The computations were performed with four parallel 2GHz processors with 3GB memory, using ILOG CPLEX version 10.0. The algorithmic procedures for SOS1 and indicators variables were utilized. A stopping time of 10 min was used for each problem, which is consistent with the maximum computational time possibly available for a reasonable implementation.

In Figure 7, we show the average number of aircraft that are infeasible for each inter-arrival time. As expected, as the number of aircraft increases, so does the expected number of infeasible aircraft. When provided with multiple flight-level options, the conflict resolution procedure is able to reduce the number of infeasible aircraft, i.e. aircraft that need to handled by a controller. In fact, just by doubling the number of available flight-levels, virtually all conflicts can be resolved through the algorithm, even at high traffic loads. Focusing on the single flight-level problem, it is observed that at traffic-levels historically experienced in ZME19, a single flight-level model can serve as an automated tool to aid air traffic controllers at current levels and beyond. This would also be consistent with providing carriers the freedom to select their preferred flight level and trajectory. Only in rare cases, would carriers be forced or suggested to take alternative trajectories.

Excluding infeasible aircraft, the average percent increase in cost per aircraft compared to optimal conflict-free flights remains low, as shown in fig. 8. The figure also demonstrates some expected results, that is, the expected cost increases with the addition of flight-levels,. This is because previously infeasible flights are forced to operate at sub-optimal flight-levels, and are included in the cost calculations. Furthermore, at the single flight level, there is a significant drop in the cost at the inter-arrival rate of 0.4, this is again related to an increase in the number of infeasible aircraft.

Also interesting in fig. 8 is the rapid jump in costs for the 2 and 3 flight-level problem for inter-arrival rates lower than 0.3. This is a powerful result that allows us to learn significantly more about airspace capacity and expected costs. This type of information, coupled with the previous infeasibility results, can be provided to a stochastic traffic flow management tool for strategic control of air traffic flow throughout the NAS that considers trade-off between capacity and fuel-expenditures. Examples of such work can be found in [17], [18]. Finally, the expected percent increase in fuel-burn over all the traffic volumes considered remains low. Even at 150 aircraft/hour (inter-arrival time of 0.4), the percent increase in costs for two flight-levels is less than 1.2 %. This is less than the fuel-burn calculated under current traffic volumes [19].

It should be noted that the structure of the sector is compatible with this type of conflict resolution. The majority of traffic is located along a few fixed lanes, which encourages the use of multiple flight-levels. Other areas that might benefit from such a structure is the Northeast corridor and Transatlantic flights.

VIII. Conclusions

A new and advanced approach based on concepts of speed control and flight-level assignment for conflict resolution over predefined routes has been presented. The developed mathematical model is formulated as a mixed integer linear program (MILP) that can be solved in less than 10 minutes, and provide near-optimal results even at high-traffic levels over multiple flight levels. The MILP model provides a broad framework for resolving conflicts through fast numerical optimization methods for large number of aircraft. The model allows for adjusting the amount of control provided, i.e. assigning arrival times, and speed commands for aircraft inside and outside the airspace. While results presented were based on a centralized model, the implementation can easily be configured to be run iteratively based on a first-come-first-served policy. We also focus on longer-term conflict resolution.

The major advancement in this paper is the formulation of a complete optimization model for speed control and flight level assignment to reduce fuel burn over time horizons between 15-45 minutes.

IX. Acknowledgments

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