

1. Consider an observational study that recorded Forced Expiratory Volume (FEV) (in 1 second) and height (in inches) measurements. The goal of the study is to investigate the association between FEV and height. Let $h_c = \text{height} - \bar{h}$ denote the mean centered height (where \bar{h} denotes the sample mean of height).

Two investigators independently analyzed the data by fitting the following models:

Investigator A: Fitted the model $E[FEV|h_c] = \beta_0 + \beta_1 h_c + \beta_2 h_c^2$ (1)

Investigator B: Fitted the model $E[FEV|\text{height}] = \delta_0 + \delta_1 \text{height} + \delta_2 \text{height}^2$ (2)

- (i) What are $E(\text{FEV}|\text{height} = \bar{h})$ in (1) and (2)? (3 points)
- (ii) How are the coefficients in the above two models related? That is, show how the δ coefficients are related to the β coefficients? **Hint:** Assume $\bar{h} = a$ – a known constant. If that helps. Start from (1) and express it as (2) to show that the β parameters are uniquely related to the δ parameters. (15 points)
- (iii) Each investigator tests the null hypothesis that the highest order term of their quadratic model would be equal to zero (versus the alternative that it is not equal to zero). Should both investigators obtain the same result? Explain. (5 points)
- (iv) What null hypothesis should be tested to answer the scientific question of interest (by investigator A)? (3 points)
- a. $H_0: \beta_0 = \beta_1 = \beta_2 = 0$
- b. $H_0: \beta_1 = \beta_2 = 0$
- c. $H_0: \beta_2 = 0$.
- (v) Both investigators A and B should obtain the same fitted values for FEV.
- a. True
- b. False
- Explain why true or why false. (3 points)
- (vi) If height is measured in centimeters (1 inch = 2.54 cm) what would be the coefficients in this new model? **Use the model in (2).** (11 points)