Problem 1 (40 points)

Imagine that you are planning your next vacations. For your trip, first you want to determine which items to pack. To take this decision, you make a list of 30 items from which you want to pick the ones to be packed. For each of these 30 items, you know its weight (in pounds) and its associated intrinsic value (the more you want to take the item, the higher its "value"), as indicated below (Table 1).

Table 1. Weight and value of each item

ITEM	Weight (pounds)	Value
Item 1	4	3
Item 2	5	2
Item 3	5	5
Item 4	9	4
Item 5	9	4
Item 6	4	4
Item 7	7	4
Item 8	3	9
Item 9	6	7
Item 10	5	7
Item 11	7	8
Item 12	8	9
Item 13	3	8
Item 14	8	3
Item 15	4	6
Item 16	3	5
Item 17	7	7
Item 18	9	8
Item 19	6	2
Item 20	7	2
Item 21	6	7
Item 22	8	2
Item 23	7	5
Item 24	9	3
Item 25	3	7
Item 26	4	4
Item 27	10	9
Item 28	4	10
Item 29	3	4
Item 30	8	6

PART I. (20 points)

Assume that for your trip you can only use a single bag. Considering the size of the items, you know **you cannot pack more than three items in this bag**. Also, due to regulations, maximum weight packed in this **bag should be 16 pounds**. Considering these constraints, calculate the optimal packing strategy (the one that maximizes the total value of the items packed)

1.1. (2 points) In the optimal solution, is item 8 packed?
 A. Yes
 B. No

1.2. (2 points) In the optimal solution, is item 10 packed?
 A. Yes
 B. No

1.3. (2 points) In the optimal solution, is item 12 packed?
 A. Yes
 B. No

1.4. (2 points) In the optimal solution, is item 18 packed?
 A. Yes
 B. No

1.5. (2 points) In the optimal solution, is item 26 packed?
 A. Yes
 B. No

1.6. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total value associated with the items that you can pack?
 A. 15
 B. 22
 C. 28
 D. 35
 E. 43
 F. 71

1.7. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total weight associated with the items that you can pack in your bag?
 A. 11
 B. 12
 C. 13
 D. 14
 E. 15
 F. 16

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PART II. (20 points)

Now, assume that for your trip you can use two bags, where you cannot pack more than **three items in the first bag, or more than six in the second bag**. Also, due to regulations, maximum weight packed are **16 pounds and 29 pounds** for the first bag and second bag, respectively. Considering these constraints, calculate the optimal packing strategy (the one that maximizes the total value of the items packed).

1.8. (2 points) In the new optimal solution, is item 8 packed?
   A. Yes
   B. No

1.9. (2 points) In the new optimal solution, is item 10 packed?
   A. Yes
   B. No

1.10. (2 points) In the new optimal solution, is item 12 packed?
   A. Yes
   B. No

1.11. (2 points) In the new optimal solution, is item 18 packed?
   A. Yes
   B. No

1.12. (2 points) In the new optimal solution, is item 26 packed?
   A. Yes
   B. No

1.13. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total value associated with the items that you can pack?
   A. 69
   B. 72
   C. 74
   D. 79
   E. 82
   F. 85

1.14. (5 points) In the optimal packing strategy (the one that maximizes the total value packed) what is the total weight associated with the items that you can pack in the first bag?
   A. 11
   B. 12
   C. 13
   D. 14
   E. 15
   F. 16

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Problem 2. (40 points +additional 20 points extra credit)

You are still planning your next vacations. For your vacations, you have decided to visit six cities (labeled as city 1, 2, ..., 6). Your trip will start and finish in your hometown (labeled as city 7). To visit each of the cities, you can either take a bus or a plane. The travel costs (in USD) and times (in hours) are shown in the tables below (Tables 2 and 3). Note that, as expected, the bus is often much cheaper, but usually takes longer. Also, not that these costs and travel times are not symmetric.

Table 2. Travel times and travel costs for bus trips

Travel time - bus (hours)

		Destination city						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Departure city	**1**	0.0	9.0	7.0	5.6	18.6	13.8	17.8
	2	9.4	0.0	12.2	13.2	22.8	14.0	18.6
	3	10.8	14.2	0.0	11.8	15.2	15.2	13.4
	4	9.0	13.4	10.2	0.0	21.0	19.0	22.6
	5	20.2	21.6	12.8	21.4	0.0	18.0	12.4
	6	14.0	13.8	14.8	21.4	14.8	0.0	8.4
	7	18.8	20.4	15.2	21.4	10.0	10.8	0.0

Travel cost - bus (USD)

		Destination city						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Departure city	**1**	0	26	48	50	44	46	38
	2	42	0	32	48	84	54	38
	3	42	60	0	38	32	32	50
	4	50	64	24	0	78	68	74
	5	76	62	38	42	0	50	56
	6	54	34	64	72	34	0	38
	7	50	50	42	76	22	26	0

Table 3. Travel times and travel costs for airplane trips

Travel time - airplane (hours)

		Destination city						
		1	2	3	4	5	6	7
Departure city	1	0.0	1.6	2.2	1.8	5.6	3.8	5.2
	2	1.8	0.0	3.2	3.8	6.0	3.4	5.0
	3	2.2	3.4	0.0	2.6	3.2	3.6	3.4
	4	2.0	3.6	2.8	0.0	5.6	5.6	5.8
	5	5.6	6.2	3.4	5.4	0.0	4.4	2.6
	6	3.6	3.6	3.4	5.6	4.0	0.0	2.4
	7	5.0	4.8	3.4	5.8	2.6	2.4	0.0

Travel cost - airplane (USD)

		Destination city						
		1	2	3	4	5	6	7
Departure city	1	0	246	324	300	688	404	652
	2	170	0	480	496	654	352	542
	3	320	446	0	394	386	402	460
	4	198	464	404	0	628	562	636
	5	560	770	350	582	0	492	260
	6	388	468	426	590	516	0	348
	7	604	624	418	698	264	362	0

PART I. (20 points)

Assume that you want to minimize your total travel time. Help: for this particular problem, this means that you will travel using only airplanes (not buses).

2.1 (5 points) What is the total travel time (in hours) for the optimal trip (the one that minimizes the total travel time)?

    A. 15.3
    B. 17.1
    C. 21.3
    D. 18.0
    E. 19.7
    F. 22.1

2.2 (5 points) What is the total cost (in USD) associated with the optimal trip (the one that minimizes the total travel time)?

    A. 2345
    B. 1892
    C. 1987
    D. 2451
    E. 2152
    F. 2213

2.3 (2 points) For the trip that minimizes the total travel time, which was the <u>first</u> city visited?

    A. City 1
    B. City 2
    C. City 3
    D. City 4
    E. City 5
    F. City 6

2.4 (2 points) For the trip that minimizes the total travel time, which was the <u>second</u> city visited?

    A. City 1
    B. City 2
    C. City 3
    D. City 4
    E. City 5
    F. City 6

2.5 (2 points) For the trip that minimizes the total travel time, which was the <u>third</u> city visited?

    A. City 1
    B. City 2
    C. City 3
    D. City 4
    E. City 5
    F. City 6

2.6 (2 points) For the trip that minimizes the total travel time, which was the <u>fourth</u> city visited?

    A. City 1
    B. City 2
    C. City 3
    D. City 4
    E. City 5
    F. City 6

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2.7 (2 points) For the trip that minimizes the total travel time, which was the <u>fifth</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

PART II. (20 points)

Assume that now you want to minimize your total travel cost. Help: for this particular problem, this means that you will travel using only buses (not airplanes).

2.8 (5 points) What is the total travel time (in hours) for the optimal trip (the one that minimizes the total travel cost)?

A. 125.3
B. 95.6
C. 204.5
D. 54.2
E. 86.4
F. 102.4

2.9 (5 points) What is the total cost (in USD) associated with the optimal trip (the one that minimizes the total travel cost)?

A. 246
B. 175
C. 254
D. 202
E. 198
F. 232

2.10 (2 points) For the trip that minimizes the total travel cost, which was the <u>first</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.11 (2 points) For the trip that minimizes the total travel cost, which was the <u>second</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.12 (2 points) For the trip that minimizes the total travel cost, which was the <u>third</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.13 (2 points) For the trip that minimizes the total travel cost, which was the <u>fourth</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.14 (2 points) For the trip that minimizes the total travel cost, which was the <u>fifth</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

PART III. (20 points) <u>Extra Credit</u>

Assume that now you want to minimize your total travel time, while guaranteeing that the total travel cost is less than or equal to 1000 USD. In this case, it is necessary to use both buses and airplanes.

2.15 (5 points) What is the total travel time (in hours) for the optimal trip (the one that minimizes the total travel time using less than or equal to 1000 USD)?

A. 42.2
B. 56.3
C. 68.2

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D. 75.2
E. 32.5
F. 48.0

2.16 (5 points) What is the total cost (in USD) associated with the optimal trip (the one that minimizes the total travel time using less than or equal to 1000 USD)?

A. 950
B. 960
C. 970
D. 980
E. 990
F. 1000

2.17 (2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the <u>first</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.18 (2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the <u>second</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.19 (2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the <u>third</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.20 (2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the <u>fourth</u> city visited?

A. City 1
B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

2.21 (2 points) For the trip that minimizes the total travel cost using less than or equal to 20 hours, which was the <u>fifth</u> city visited?

A. City 1
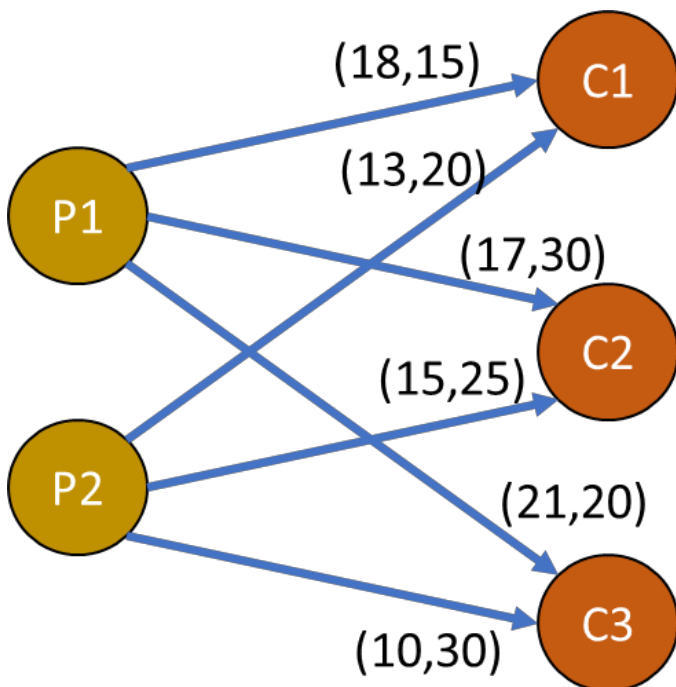B. City 2
C. City 3
D. City 4
E. City 5
F. City 6

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Problem 3. (20 points)

Suppose that you are about to open a company that produces paint. You have built two production plants (P1 and P2) to supply the demands of three nearby cities (C1, C2, and C3), but you still need to decide how much production capacity (in tons) you should assign to each plant. Even though you haven't decided the production capacity of each plant, due to contractual obligations the production capacity of P1 must be at least 60% of the production capacity of P2.

The figure below indicates the expected demand in each city (in tons). Also, the figure indicates (in parenthesis) the unitary flow cost (dollars per ton) and the maximum capacity (in tons) associated with each arc (respectively).

3.1. (5 points) Considering the information given, how much production capacity should be given to plant P1, so that the final paint distribution cost is minimized?

    A. 5
    B. 15
    C. 30
    D. 45
    E. 50
    F. 65

3.2. (5 points) Considering the information given, how much production capacity should be given to plant P2, so that the final paint distribution cost is minimized?

    A. 5
    B. 15
    C. 30
    D. 45
    E. 50
    F. 65

3.3. (10 points) What is the optimal distribution cost for this problem?

    A. 755
    B. 885
    C. 935
    D. 1065
    E. 1115
    F. 1205

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