Problem 1: Growth Accounting

In the early 1990s, the high growth rates experienced in the Hong Kong and Singapore led these economies to be dubbed as part of the East Asian Growth Miracle. Looking at these economies, a student argues that all that is needed for rapid continuous growth is heavy investment in capital and maintaining a high capital to output ratio. In this question, you will investigate if this statement is true.

a Download the dataset `hkg_sgp_data.xls` from Canvas. Calculate GDP per capita and capital per person for each of the two economies in the dataset for the years 1960-2014. You do not need to print out or submit your answer for part a).

b For each economy, do a scatterplot of the growth rate in GDP per capita against the capital to output ratio. What trend do you observe?

c Let's try to understand the trend in part b). Consider the following production function, $Y = zK^{\alpha}N^{1-\alpha}$. Note here I am identifying population N with labor supply. First write down what output per capita, $y = Y/N$ is in terms of z, k, α where $k = K/N$. Suppose z is constant. At low levels of k, is the rate of change in output per capita wrt capital per capita, i.e. $\frac{dy}{dk}$ small or large? What about at high levels of k?

d Now let's do some growth accounting. We want to understand the percentage of growth that can be explained by growth in k. Observe that we can write $\ln y = \ln z + \alpha \ln k$. Recall that we can measure the growth rate in GDP per capita with the change in logs, i.e. $g_y = \ln y_{t+1} - \ln y_t$. Using your production function, back out the growth rate in z for each economy. In a table, report the average growth rates of y, k, z for the periods 1960-1969, 1970-1979, 1980-1989, 1990-1999, 2000-2009 and for the years ≥ 2010. You may assume $\alpha = 1/3$.

e What percentage of growth can be explained by capital growth for each of those time periods? What percentage of growth can be explained by z for each of those time periods? Together with your answers in part b) and c), do you think the student's statement is true?

Problem 2: Golden Rule of Saving

Many of the East Asian economies followed a pattern of high savings rate and heavy investment to grow their economies. Increases in the savings rate have an ambiguous effect on long run consumption per person: they raise per capita capital and output, but reduce the share of that output that is consumed. We want to analyze this tradeoff. To do so rather than analyzing the balanced growth implied by a given model of savings behavior, we'll try and figure out the savings behavior that gives rise to a desirable balanced growth path.

The analysis you will need to follow will be a little different from our class room analysis of the Solow model analysis in two regards. First, you will study an economy in which there is population growth, but not TFP growth. We have done things the other way around in class. The Solow model implies in such situations that in steady state GDP grows at the same rate as population, but GDP per capita does not grow. Second rather than deriving or assuming a saving rate and obtaining its implications for balanced growth, you will go in the reverse direction and solve for a balanced growth path/steady state and then ask: what savings rate supports this?

Here are the details. Assume an economy with a Cobb-Douglas production technology $Y = zK^{\alpha}N^{1-\alpha}$ in which TFP is constant, everyone works, population grows at a rate n, i.e. $\frac{N_{t+1}}{N_t} = 1 + n$ and depreciation is δ. Now complete the following steps.

1. Explain why in such an economy:

$$C_t + K_{t+1} - (1 - \delta)K_t = zK_t^{\alpha}N_t^{1-\alpha}, \tag{*}$$

   where C_t equals aggregate consumption. Relate this to the accounting identity: $Y = C + I + G + NX$.

2. Assume that the economy is on a balanced growth path in which capital, consumption, and output are growing at the same rate as population. (Note: TFP is assumed to be constant in this part of the question and its growth is zero). Assume that everyone works. Use the previous equation (*) to show that on this balanced growth path:

$$c = zk^{\alpha} - (n + \delta)k$$

   where lower case letters c and k denote *per capita* variables. (In doing this, think carefully about the K_{t+1} variable in (*)).

3. Maximize the right hand side of the previous equation over k to find the balanced growth path capital per capita value that maximizes balanced growth path per capita consumption.