Homework 2

Introduction to Bayesian Econometrics

Winter term 2022/23

PROBLEM 1. Year-on-year monthly inflation rates are defined as $\pi_t = 100 \times \log(\Pi_t / \Pi_{t-12})$ where Π_t is the consumer price index (CPIAUCSL) in the United States at time $t = 1, \ldots, T$. Consider an AR(1) model of the form:

$$\pi_t = \mu + \phi \pi_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2).$$

Assume an inverse Gamma prior on the error variances, $\sigma^2 \sim \mathcal{G}^{-1}(1, 1)$ and a Gaussian prior on the parameter $\mu \sim \mathcal{N}(0, 10)$. For the persistence parameter, implement the following priors:

- a Gaussian prior, $\phi \sim \mathcal{N}(0, 10)$, i.e. $\phi \in \mathbb{R}$;

- a Beta prior, $\phi \sim \mathcal{B}(a_0, b_0)$, i.e., $\phi \in (0, 1)$, with shape parameters a_0 and b_0 that you may choose in an adequate fashion.

Download and transform the most recent series (e.g., from fred.stlouisfed.org), and estimate the model for these two priors. Briefly describe how you implemented your MCMC algorithm and mention key sampling steps. Discuss your findings in terms of the posteriors for ϕ.

Specifically consider the following aspects: (1) what is the posterior probability of $|\phi| < 1$ for the unrestricted Gaussian prior; (2) in light of your answer to the first point, is the imposed restriction on the support of ϕ based on the Beta prior useful; (3) discuss the efficiency of your MCMC algorithm for the two priors.

PROBLEM 2. Consider the model from PROBLEM 1, but now introduce a sample split at time t_0. In particular, assume a model of the form:

$$\pi_t = \begin{cases} \mu_0 + \phi_0 \pi_{t-1} + \sigma_0 \epsilon_t & \text{for } t = 1, \ldots, t_0, \\ \mu_1 + \phi_1 \pi_{t-1} + \sigma_1 \epsilon_t & \text{for } t = t_0 + 1, \ldots, T \end{cases}, \quad \epsilon_t \sim \mathcal{N}(0, 1).$$

Restrict your sample to the period from January 1965 to December 2006. The splitting month at t_0 is December 1985, i.e., this roughly splits your sample into two equally sized periods which approximately correspond to those commonly referred to as the "Great Inflation" and the "Great Moderation."

Estimate your model using the unrestricted Gaussian prior and discuss the following aspects in detail: (1) compute the probabilities of $|\phi_0| < 1$ and $|\phi_1| < 1$ and compare; (2) is there evidence for a change in the persistence of inflation over time; (3) was there a significant shift in the *unconditional* mean and volatility of inflation? Exclude non-stationary draws from your posterior for answering (3).

Remark: If you are interested in related research, the paper "Were There Regime Switches in U.S. Monetary Policy?" by C.A. Sims and T. Zha, *American Economic Review* **96**(1), p. 54–81, 2006, may be of relevance.