

IB2070

## Mathematical Programming II

### Individual Assignment [12 CATS], 2022

#### Assignment Instructions

All assignments must be submitted ONLINE via my.wbs by **7pm UK time** on the date displayed against this assessment.

Please ensure that you have inserted a completed [assignment coversheet](#), which must be included as the first page of your script. This should include your Student ID number, but not your name.

#### Word Limit

1500 words

#### Word Count Policy

WBS has a school-wide policy on word counts. This is strictly enforced to ensure consistency across modules and programme. You can find more information about this policy in your Student Handbook under Academic Practice - [4i. Word count policy](#).

**This is a strict limit not a guideline: any piece submitted with more words than the limit will result in the excess not being marked.**

#### Academic Practice

Please ensure you read the full guidelines for [Academic Practice](#) in the Undergraduate Handbook and ensure you understand it. **If in doubt, please seek clarification in advance of your submission.** This includes important information on:

- Cheating, plagiarism and collusion
- Correct referencing
- Using internet sources in assessments
- Academic writing
- English Language support
- Word count policy

When you submit this assignment online, you will be required to tick a declaration box indicating that the work involved is entirely your own. Each assignment will be put through plagiarism software to identify any collusion or inadequate referencing of materials used from different sources.

We would consider taking action if your work:

1. is too reliant on the words of particular authors (rather than presenting your ideas in your own words), if the essay uses the ideas or words of an author without referencing them or putting their words into quotations (**plagiarism**).
2. suggests that you have worked very closely with another student or students (unless explicitly asked to do so by your Module Leader/Tutor) (**collusion**).
3. includes unreferenced work that you have previously submitted for any accredited course of study (unless explicitly asked to do so by your Module Leader/Tutor) (**self-plagiarism**).

### **Extensions and Self-certification**

Late submissions will incur a penalty of 5% for every 24-hour period after the due date and time, i.e. this begins one minute after the submission deadline (beginning at 7.01pm).

Requests for **specific extensions** (of up to 15 days) which are typically for longer and more serious concerns must be submitted via my.wbs ideally 72 hours BEFORE the deadline. Extensions can only be approved if you clearly detail your circumstances and provide supporting documentation (or a reason as to why you cannot provide the supporting documentation at the time) as set out in the [Mitigating Circumstances Policy](#).

**Self-certification** is a university-wide policy whereby you are permitted an automatic extension of 5 working days on eligible written assessed work without the need for evidence. WBS permits self-certification for all types of written, assessed works such as essays and dissertations. It is not permitted for exams, course tests, or presentations.

You can self-certify twice within each year of study, starting from the anniversary of your course start date. This will cover all eligible written assessments that fall within the self-certification period, as long as they have not previously had an extension applied. To find out further details about the self-certification policy please see: <https://my.wbs.ac.uk/-/academic/20778/item/id/1244460/>.

If you wish to self-certify for an extension of 5 working days, please select 'Self-certification' in the Extension Type field. If you wish to request a longer extension than 5 working days, please leave the Extension Type as 'Standard'.

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Your assignment instructions begin below.

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# IB2070 Mathematical Programming II

## ASSIGNMENT [12 CATS], 2022-23

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### Instructions: PLEASE READ CAREFULLY!

- The assignment is based on the knowledge of the first four weeks of the module. It is for **individual** work. Any similarity between different submitted works will be investigated for plagiarism according to the University's policy.
  - Handwritten solutions will **not** be accepted. Please write your answers clearly using LaTeX or MS Word with proper mathematical notation and font size 11 or 12 and with sufficient margins. Then convert it to a PDF file. Make sure that each sheet has your University ID Number and the question number(s). Your work should **not** exceed **4 pages**, which should **not** include any screenshot involving formulation and solution with software. An appendix is **not** expected, which will not necessarily be read if any.
  - Submit your work online (in PDF format) by **19:00 (UK time), Thursday, 24<sup>th</sup> November 2022** via **my.wbs**. No other submission method will be accepted. Penalty for late submissions applies automatically.
  - Please note that correctness of mathematics alone is not sufficient. Detailed explanations of your working as well as complete and clear presentation of your solution **and** solution process are equally important.
  - An online forum will be open for any requests for clarification on the assignment, but no requests will be accepted within 24 hours of the submission deadline.
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Consider the following scheduling problem of parallel machines: A set  $\{J_1, \dots, J_n\}$  of  $n$  jobs need to be scheduled onto  $m$  ( $m > 1$ ) parallel machines  $M_1, \dots, M_m$ . The time requirements for processing these jobs are, respectively,  $p_1, \dots, p_n > 0$ . Each of these  $n$  jobs needs to be processed without interruption on one (and only one) of these  $m$  machines. Unless specified otherwise, all machines are continuously available from time 0. The objective is to assign all the jobs to the machines so as to minimize the *makespan*, i.e., the time point at which all jobs are finished.

### Question 1 [40%]

Suppose that all  $m$  machines are identical.

- (a) Formulate the problem as a binary integer linear program (ILP). Provide all the details of your formulation, including clear definition of your variables, explanations of the objective function and of each constraint.

- (b) Let  $m = 10$  and the processing requirements of  $n = 100$  jobs be given in the attached Excel file, "Scheduling\_Instance.xlsx". Solve the LP relaxation of your binary ILP formulated in part (a) and *present* your optimal LP solution. Based on the optimal LP solution, suggest at least five *feasible* solutions to the original ILP. Explain the reasons behind your suggestion. What are your suggested five feasible job assignments? What are the corresponding makespans of the five schedules? (It is recommended to present some of your answers in a tabular form.)

### Question 2 [30%]

Suppose that the  $m$  machines may have different speeds  $1 = s_1 \leq \dots \leq s_m$ . In other words, the processing time of job  $J_j$  on machine  $M_i$  is  $p_j/s_i$ .

- (a) Do the same as in Q1(a) for a binary ILP formulation.
- (b) Do the same as in Q1(b) except that the ILP is the one you have obtained in Q2(a) with machine speeds  $s_1 = s_2 = s_3 = 1$  and the remaining 7 speeds given as the reordered 7 digits of your student ID number with possible 0 replaced by 1. (For example, if your student ID number is 2092444, then your 10 machine speeds are 1, 1, 1, 1, 2, 2, 4, 4, 4, 9.)

### Question 3 [30%]

Under the same setting as in Question 2 except that the last machine  $M_m$  is unavailable due to maintenance during *time period*  $[a, b]$  for some parameters  $a, b$  with  $0 < a < b$ .

- (a) Do the same as in Q2(a) for a binary ILP formulation.
- (b) Do the same as in Q2(b) with  $a = 10^9$  and  $b = 2a$ .

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(END OF ASSIGNMENT)