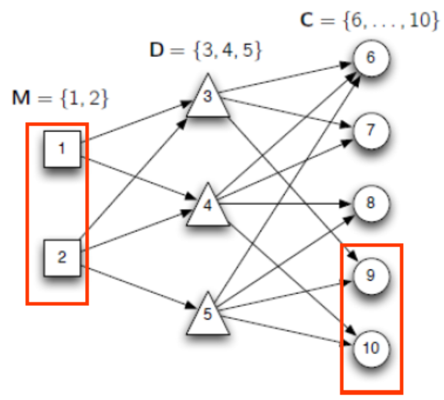
Billets are transported from 2 different Billet Mills ($\mathbf{M}$ = {1, 2}) to distribution centres ($\mathbf{D}$ = {3, 4, 5}). These distribution centres serve five customers ($\mathbf{C}$ = {6, ..., 10}).

Billet Mill one can produce up to 260 billets, Billet Mill two has a capacity of 180 billets. The yield at Billet Mill one is $N(0.85, 0.1)$, while the yield at Billet Mill two is $N(0.82, 0.05)$.

Customer 6, 7 and 8 require 25 billets each, but customer 9 requires $N(115, 10)$ billets and customer 10 needs $N(100,15)$.

To meet this demand. **What number of billets can flow through each arc at the least possible cost? Hint: Use Expected Value Optimization technique.**



Assume a cost price to be R16 per arc.

A graph $G = (V, E)$ with $E = \{(i, j) | i, j \in V\}$

$p_i, i \in M$

$d_i, i \in C$

$c_{ij}, \quad i, j \in V \mid (i, j) \in E$

x_{ij} ≜ the number of billets transported from $\boldsymbol{i}$ to $\boldsymbol{j}$, where $\boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{V} \mid (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{E}$

$$\min z = \sum_{(i,j)\in\mathbf{E}} c_{ij} x_{ij}$$

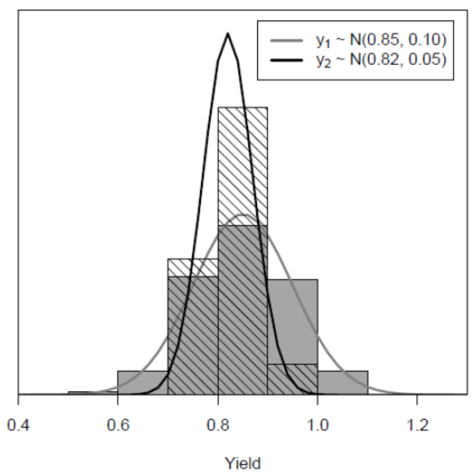
subject to

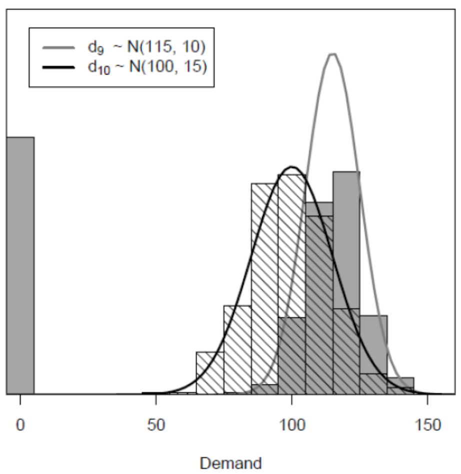
$$\sum_{j\in\mathbf{D}|(i,j)\in\mathbf{E}} \frac{x_{ij}}{\tilde{y}_i} \le p_i \qquad \forall i \in \mathbf{M}$$

$$\sum_{i\in\mathbf{M}|(i,j)\in\mathbf{E}} x_{ij} \ge \sum_{k\in\mathbf{C}|(j,k)\in\mathbf{E}} x_{jk} \qquad \forall j \in \mathbf{D}$$

$$\sum_{j\in\mathbf{D}|(j,k)\in\mathbf{E}} x_{jk} \ge \tilde{d}_k \qquad \forall k \in \mathbf{C}$$

$$x_{ij} \ge 0 \qquad \forall (i,j) \in \mathbf{E}$$





$p_1 = 260$

$p_2 = 180$

$\tilde{y}_1 \sim N(0.85, 0.10)$

$\tilde{y}_2 \sim N(0.82, 0.05)$

$d_6 = 25$

$d_7 = 25$

$d_8 = 25$

$\tilde{d}_9 \sim N(115, 10)$

$\tilde{d}_{10} \sim N(100,15)$