

**PART I: COMPUTER COMPONENT (Each part of each problem below is worth 1 mark for a total of 15 marks)**

All your solutions for the Computer Component are to be saved in ONE Word file that, when completed, can be subsequently saved and submitted as **ONE pdf file**.

At this point, create this Word solutions file and save under the filename:

“Your last name\_Math\_Final\_ComputerComponent ”

**PROBLEM 1**

**Problem 1a: Create Your Own Hypothetical Customer Experience/Satisfaction Questionnaire Survey**

In your Word solutions file type in a main heading entitled “Final Computer Portion Version A\_1”

Below this heading, type in the subheading “Problem 1a”.

In your Word solutions file, under the subheading “Problem 1a” create your own survey questionnaire similar in Format to Figure 1 Hypothetical Survey Questionnaire (see below). You may find it easier to create this survey within a table. In your survey questionnaire, proceed to replace all the items shown in Figure 1 that are currently in red font, based on the following specifications:

You are to create your own hypothetical customer experience/satisfaction questionnaire survey based on the format shown in Figure 1 below, consisting of 2 qualitative variables and 4 quantitative variables that satisfy the following specifications:

All survey questions (variables) are to relate to the same specific hypothetical population of your choice (ie.”regular customers of John Smiths Seafood Restaurant”). Your survey can focus on customers relating to a hypothetical company or government organization offering a hypothetical product ( or products) and/or service (or services). Make sure you create your own hypothetical specified population and do not use the same “names etc.” as used in the surveys presented in the computer part of this course or elsewhere. **Avoid plagiarism.**

- Your **first variable** is to be a binary nominal variable ( with two distinct values) with assigned numeric codes “1” and “2”.
- Your **second variable** is to be an ordinal variable with three distinct values with assigned numeric codes “1”, “2” and “3” .
- Your **third and fourth variables** are to be continuous variables that describe specific characteristics relating to your population. You should attempt to create variables that could be considered to be **independent** variables that help predict or explain your fifth and sixth variables (see below)
- Your **fifth and sixth variables** are to be quantitative variables (either discrete or continuous). You should attempt to create variables that could be considered **dependent** on your third and fourth variables. In a typical customer experience/satisfaction survey these variables would attempt to measure customer attitudes or behaviour such as: the level of customer satisfaction or the degree to which the customer uses or interacts with the specific product(s) or service(s) or company defined in the survey etc..

**NOTE 1:** Marks will be deducted, if you do not create a **customer survey**, with the types of variables as described above.

**NOTE 2:** In ALL your solutions relating to ALL questions relating to Confidence Intervals, Tests of Hypotheses, and Regression, **make sure that you finish your solution with a practical interpretation stated in terms of the context of the survey you created.**

Note: It is okay if some (but not all) of the variables that you use in your survey are variables similar to ones used in the surveys presented in the computer part of this course or elsewhere. You must avoid creating a survey where all variables and all the values for each of the variables are identical to a survey presented in the computer part of this course or elsewhere. In other words, **do NOT plagiarize**.

**Problem 1b: Create a Set of 20 Hypothetical Responses**

In your Word solutions file, under Figure 1, type in the subheading: “Problem 1b”.

In your Word solutions file under the subheading Problem 1b, create a table similar in format to Figure 2 “Twenty Hypothetical Survey Responses: Sample Data” (see below). In your table proceed to replace all the items in Figure 2 that are currently in red font, based on the following specifications.

You are to create a set of 20 hypothetical numeric responses for each of the six variables in your questionnaire survey according to the following specifications:

- For all the responses, type in numeric values. You will be asked to recode some of these variables in a subsequent problem.
- All numbers should be whole numbers (no decimals, no fractions, no commas, no dollar signs etc)
- For each of your Variables 3, 4, 5, 6, create a diverse set of numbers that is roughly symmetrically distributed, where close to half the numbers are below the mean and the other half of the numbers are above the mean.

(See Figure 1 below:)

**Figure 1:Hypothetical Survey Questionnaire**

Questionnaire		For Office Use Only	
Please check off those boxes that apply to you.		Variable	Code
1. Create Survey Question? (binary nominal variable)		Var Name?	
<input type="checkbox"/> Name this value?			1
<input type="checkbox"/> Name this value?			2
2. Create Survey Question? (ordinal variable with 3 values)		Var Name?	
<input type="checkbox"/> Name this value?			1
<input type="checkbox"/> Name this value?			2
<input type="checkbox"/> Name this value?			3
3 Create Survey Question? (continuous independent variable)		Var Name?	
4.Create Survey Question? (continuous independent variable)		Var Name?	
5. Create Survey Question? (continuous or discrete dependent variable)		Var Name?	
6. Create Survey Question? (continuous or discrete dependent variable)		Var Name?	

(See Figure 2 below:)

**Figure 2: Twenty Hypothetical Survey Responses: Sample Data**

Response No.	<i>Var 1 Name?</i>	<i>Var 2 Name?</i>	<i>Var 3 Name?</i>	<i>Var 4 Name?</i>	<i>Var 5 Name?</i>	<i>Var 6 Name?</i>
1	—	—	—	—	—	—
2	—	—	—	—	—	—
3	—	—	—	—	—	—
4	—	—	—	—	—	—
5	—	—	—	—	—	—
6	—	—	—	—	—	—
7	—	—	—	—	—	—
8	—	—	—	—	—	—
9	—	—	—	—	—	—
10	—	—	—	—	—	—
11	—	—	—	—	—	—
12	—	—	—	—	—	—
13	—	—	—	—	—	—
14	—	—	—	—	—	—
15	—	—	—	—	—	—
16	—	—	—	—	—	—
17	—	—	—	—	—	—
18	—	—	—	—	—	—
19	—	—	—	—	—	—
20	—	—	—	—	—	—

**Problem 1c Create a StatCrunch Data File for your Sample Data**

Create a StatCrunch data file consisting of the set of 20 numeric responses that you created in Problem 1b. Save this StatCrunch data file under the name “Your last name\_final\_data” by selecting the menu options: **Data, Save**. Note that this saves the data file to your Mydata folder at the Statcrunch website.

**Problem 1d Recode and Resave the Qualitative Variables in your StatCrunch Data File**

Before analyzing the qualitative variables in your StatCrunch data file, you will recode the responses to these variables to recognizable text descriptions of your choosing. When you recode each qualitative variable, make sure that you insert the recoded values in new columns (instead of replacing the original columns). Resave your StatCrunch data file under the same name “Your last name\_final\_data” (i.e. overwrite the original file).

**Note:** Once you have completed this exam, it is very important that you submit this data file, along with the other solution files.

To submit this data file, you will need to save this file to your hard drive, at which point you can upload this file when submitting your solutions files for marking.

To save this StatCrunch data file to your hard drive, with the data file open in StatCrunch, **click the menu options: Data, Export.** Make sure that in the Export to computer window, all variables are selected and that “comma” is selected in the Delimiter box. Then select the Export button.

This will save your data file, in the form of a comma delimited csv file, to the Downloads folder on your hard drive of your computer. This will be the file that you submit with your solutions files for marking.

**NOTE: Your solutions will NOT be marked unless you submit this data file, in the form of a comma delimited csv file as described above.**

To complete Problem 1d, open the exported data file and it should open in Excel or some other spreadsheet file.

Copy and paste all the variables and data from the Excel (spreadsheet) file to your Word solutions file under the subheading Problem 1d. This will ensure that your columns are properly aligned when pasted to your Word solutions file.

#### **Problem 1e.**

In this question you are to use StatCrunch to construct a 99% confidence interval for the population mean relating to Variable 3 based on the twenty values you created for this variable in Problem 1 above. Copy and paste the StatCrunch Confidence Interval window (which displays the answer) to your Word solutions file under the subheading Problem 1e. Under this window, in the Word solutions document, type in the confidence interval computed. Interpret your solution. Appropriately interpret your interval.

#### **Problem 1f**

Referring to Problem 1e above, use StatCrunch to determine what sample size is required for estimating the population mean relating to Variable 3 assuming a margin of error equal to 10% of the sample mean and a 99% level of confidence? To answer this, assume that the population standard deviation equals the sample standard deviation, for just this one problem part 1f. Copy and paste the StatCrunch window (which displays the answer) to your Word solutions file under the subheading Problem 1f. Under this window, in the Word solutions document, type in the required minimum sample size.

#### **Problem 1g**

In constructing the confidence interval in Problem 1e above, you made a key assumption regarding the sample data for Variable 3. At a 2% level of significance, conduct the appropriate hypothesis test to check this assumption. In your Word solution file, under a subheading entitled Problem 1g, display all the steps using the 4 step P-value Approach. Copy and paste the Hypothesis Test results window from StatCrunch to your Word solutions file under Step 2 of the test. Interpret your final decision.

**Problem 1h**

At a 5% level of significance, use StatCrunch to conduct the appropriate test of hypothesis for the true population proportion of Variable 1's customers that have the attribute represented by the "2" code, based on the 20 data values you created for Variable 1 in your survey in Problem 1. In this problem you are to test whether the population proportion is less than 50%. In your Word solution file, under a subheading entitled Problem 1h, display all the steps using the "4 step P-value Approach". Copy and paste the Hypothesis Test Results window (that displays the correct test statistic and P-value) from StatCrunch to your Word solutions file under Step 2 of the test. Interpret your final decision.

**Problem 1i**

At a 1% level of significance, conduct a two-tailed test of hypothesis comparing the equality of two population means defined as follows. The first population mean relates to the mean of those values of Variable 4 that have the attribute represented by the "1" code for Variable 1. The second population mean relates to the mean of those values of Variable 4 that have the attribute represented by the "2" code for Variable 1. Do NOT use the pooled variance option when conducting this test and assume normal populations. In your Word solution file, under a subheading entitled Problem 1i, display all the steps using the 4 step P-value Approach. Copy and paste the Hypothesis Test results window from StatCrunch to your Word solutions file under the under Step 2 of the test. Interpret your final decision.

**Problem 1j**

In conducting the Two Population Means test in Problem 1i above, you made the assumption of not using the pooled variance option. At a 5% level of significance, test this assumption. In your Word solution file, under a subheading entitled Problem 1j display all the steps using the 4 step P-value Approach. Copy and paste the appropriate Hypothesis Test results window from StatCrunch to your Word solutions file under Step 2 of the test. Interpret your final decision.

**Problem 2****Problem 2a:**

Find the equation of the linear regression line relating Variable 6 and Variable 4 in your survey, based on the 20 pairs of sample data values you created for these two variables in Problem 1. Copy and paste the first screen of the Simple Linear Regression results window from StatCrunch to your Word solutions file under the subheading Problem 2a. Type the linear regression equation under the pasted Simple Linear Regression results window.

**Problem 2b:**

Based on the appropriate output displayed in the Simple Linear Regression Results window, what percent of the variation in the dependent variable is explained by the independent variable? Type your answer in the Word solutions file under the subheading Problem 2b. Appropriately Interpret your answer.

**Problem 2c**

Based on the appropriate output displayed in the Simple Linear Regression Results window, is there a positive (direct) or negative (inverse) relationship between the independent variable and the dependent variable. How can you tell? Explain by referring to at least two different sources of output displayed in the Simple Linear Regression Results window. Type your answer in the Word solutions file under the subheading Problem 2c. Make sure that your answer is stated in the context of your survey.

**Problem 2d**

Based on the appropriate output displayed in the Simple Linear Regression Results window, conduct the appropriate hypothesis test to see if there is a significant positive relationship between the independent variable and the dependent variable. Assume a Level of Significance of 5%. In your Word solution file, under a subheading entitled Problem 2d, display all the steps using the 4 step P-value Approach. Type the appropriate P-value in the Word solutions file, under Step 2 of the hypothesis test. Appropriately interpret your final decision.

**Problem 2e**

Construct a 99% prediction interval for Variable 6 based on a specific value that you specify for Variable 4. Based on the appropriate Simple Linear Regression Results window, pasted in your Word document, type the prediction interval in the Word solutions file under the subheading Problem 2e. Appropriately interpret your prediction interval.

## **PART II: THEORY COMPONENT: (total 60 marks)**

In order to get full marks for any solution, you need to SHOW ALL WORK and clearly show all the correct formulae with all the numbers correctly substituted in the correct formula. It is assumed that you are using your calculator in all computations unless otherwise specified in the question. Where relevant, keep all your work and your final answer to a minimum of four decimals, unless otherwise stated.

**NOTE: In ALL your solutions relating to ALL questions in the Theory Component, make sure that you finish your solution with a practical interpretation or explanation stated in terms of the context of the survey you created.**

Note that all the statistics tables (z, t-table etc..) are provided in Appendix 1 at the end of this exam.

For this Theory Component, it is assumed that you will print out this component and that you will write in all your work in the space provided in each question. Once all your solutions have been completed, scan all the solutions for the Theory Component into ONE PDF file entitled:

**“Your last name Math\_Final\_Theory\_Component**

**If you do not print out this component it is very important that you submit each of your solutions in a way so that the question displays above each solution.**

### **Problem 3**

#### **Problem 3a**

Construct a 96% confidence interval estimate for the true population proportion of Variable 1's customers that have the attribute represented by the “1” code, based on the 20 data values you created for Variable 1 in your survey in Problem 1. Interpret your confidence interval. [6 marks]



**Problem 3b**

Referring to Problem 3a above, find the minimum number of customers that must be sampled in order for the 96% confidence interval estimate to be within 2% of the true population proportion. Use the sample proportion from your survey in Problem 1, as the preliminary estimate of the true population proportion. Show all work [3 marks]

**Problem 3c.** What key assumption did you make in order to be able to use the formula shown in Problem 3a above? Is this assumption satisfied based on the 20 values you created for Variable 1 in Problem 1? Fully Explain. [2 marks]

**Problem 3d.** In general, what factors can affect the margin of error of the confidence interval estimate for a population proportion? Fully explain your answer. [3 marks]

**Problem 4****Problem 4a**

At a 1% level of significance, conduct the appropriate test of hypothesis involving the population mean for Variable 5 in your survey. Create a right tailed test where you specify the hypothesized mean (feel free to specify any value for the population claim that makes sense given your data). In your answer make sure that you show  $H_0$  and  $H_a$ , sketch the sampling distribution showing the critical value(s) and the rejection region. Use your calculator to compute the test statistic and also state the decision and conclusion in the context of the survey. Show all your work . [10 marks] .

**Problem 4b** What key assumption did you make in conducting the test in Problem 4a. Fully Explain.

[2 marks]

**Problem 5**

At a 2.5% level of significance, conduct the appropriate hypothesis test to see if there is a statistically significant positive correlation between Variable 3 and Variable 5, based on the twenty 20 pairs of sample data values you created for these two variables in Problem 1. In your answer make sure that you show  $H_0$  and  $H_a$ , sketch the sampling distribution showing the critical value(s) and the rejection region. Use your calculator to compute the correlation coefficient and test statistic and also state the decision and conclusion in the context of the survey. Show all your work . [10 marks]



**Problem 6**

At a 1% level of significance conduct a test of hypothesis to determine whether there is a significant relationship between Recode(Variable1) and Recode(Variable2), based on the twenty customer responses you created in Problem 1. In your answer make sure that you construct, by hand, the appropriate table showing all the frequencies, expected, and marginal frequencies. Also show  $H_0$  and  $H_a$ , sketch the sampling distribution showing the critical value(s) and the rejection region. Use your calculator to make all the required computations including the test statistic and also state the decision and conclusion in the context of your survey. Show ALL work.

[10 marks]



**Problem 7****Problem 7a**

At a 2 % level of significance, conduct a two-tailed test of hypothesis comparing the equality of two population means defined as follows. The first population mean relates to the mean of those values of Variable 6 that have the attribute represented by the “1” code for Variable 1. The second population mean relates to the mean of those values of Variable 6 that have the attribute represented by the “2” code for Variable 1. Use the pooled variance option when conducting this test. In your answer make sure that you show  $H_0$  and  $H_a$ , sketch the sampling distribution showing the critical value(s) and the rejection region. Use your calculator to make all the required computations and also state the decision and conclusion in the context of your survey. Show all your work

[10 marks]

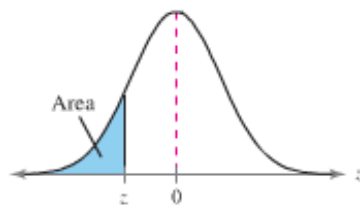




**Problem 7b** What assumptions did you make in conducting the test in Problem 7a above. Identify ALL key assumptions and fully explain. [4 marks]

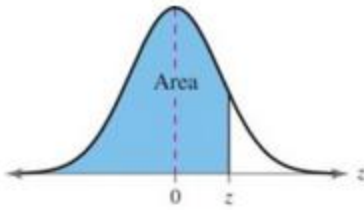
## APPENDIX 1

**Table 4— Standard Normal Distribution**

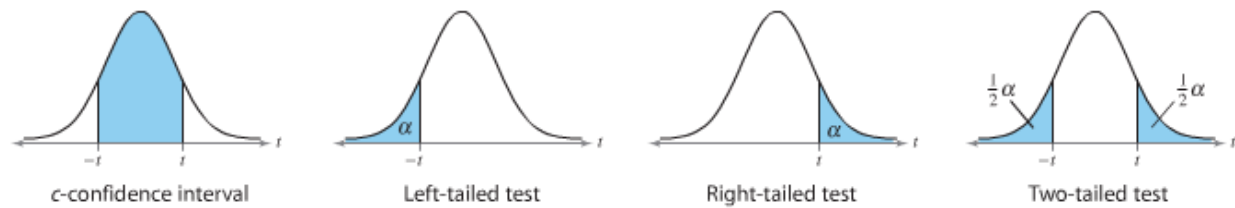


<b>z</b>	<b>.09</b>	<b>.08</b>	<b>.07</b>	<b>.06</b>	<b>.05</b>	<b>.04</b>	<b>.03</b>	<b>.02</b>	<b>.01</b>	<b>.00</b>
− 3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
− 3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
− 3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
− 3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
− 3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
− 2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
− 2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
− 2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
− 2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
− 2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
− 2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
− 2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
− 2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
− 2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
− 2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
− 1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
− 1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
− 1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
− 1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
− 1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
− 1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
− 1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
− 1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
− 1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
− 1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
− 0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
− 0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
− 0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
− 0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
− 0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
− 0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
− 0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
− 0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
− 0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
− 0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

**Table 4 Standard Normal Distribution (continued)**

[illegible]

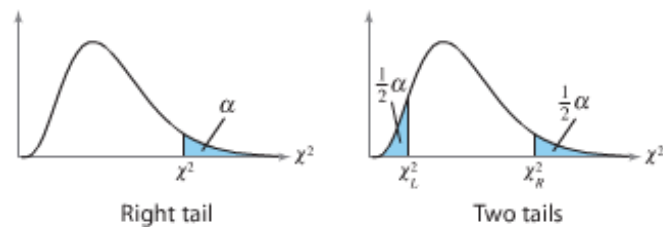
**Table 5— t-Distribution**



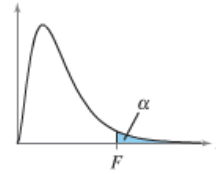
Level of confidence, $c$		0.50	0.80	0.90	0.95	0.98	0.99
One tail, $\alpha$		0.25	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, $\alpha$	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		.816	1.886	2.920	4.303	6.965	9.925
3		.765	1.638	2.353	3.182	4.541	5.841
4		.741	1.533	2.132	2.776	3.747	4.604
5		.727	1.476	2.015	2.571	3.365	4.032
6		.718	1.440	1.943	2.447	3.143	3.707
7		.711	1.415	1.895	2.365	2.998	3.499
8		.706	1.397	1.860	2.306	2.896	3.355
9		.703	1.383	1.833	2.262	2.821	3.250
10		.700	1.372	1.812	2.228	2.764	3.169
11		.697	1.363	1.796	2.201	2.718	3.106
12		.695	1.356	1.782	2.179	2.681	3.055
13		.694	1.350	1.771	2.160	2.650	3.012
14		.692	1.345	1.761	2.145	2.624	2.977
15		.691	1.341	1.753	2.131	2.602	2.947
16		.690	1.337	1.746	2.120	2.583	2.921
17		.689	1.333	1.740	2.110	2.567	2.898
18		.688	1.330	1.734	2.101	2.552	2.878
19		.688	1.328	1.729	2.093	2.539	2.861
20		.687	1.325	1.725	2.086	2.528	2.845
21		.686	1.323	1.721	2.080	2.518	2.831
22		.686	1.321	1.717	2.074	2.508	2.819
23		.685	1.319	1.714	2.069	2.500	2.807
24		.685	1.318	1.711	2.064	2.492	2.797
25		.684	1.316	1.708	2.060	2.485	2.787
26		.684	1.315	1.706	2.056	2.479	2.779
27		.684	1.314	1.703	2.052	2.473	2.771
28		.683	1.313	1.701	2.048	2.467	2.763
29		.683	1.311	1.699	2.045	2.462	2.756
$\infty$		.674	1.282	1.645	1.960	2.326	2.576



**Table 6 — Chi-Square Distribution**



Degrees of freedom	$\alpha$									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



**Table 7— F-Distribution**

d.f. <sub>D</sub> : Degrees of freedom, denominator	$\alpha = 0.01$														
	d.f. <sub>N</sub> : Degrees of freedom, numerator														
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	4.37	14.20	14.02	13.93	13.84
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39

Table 7— *F*-Distribution (continued)

d.f. <sub>D</sub> : Degrees of freedom, denominator	$\alpha = 0.05$														
	d.f. <sub>N</sub> : Degrees of freedom, numerator														
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84