

1.
  - a) Explain how you would perform ANOVA on a single replicate  $2^{6-2}$  design. 5 marks
  - b) Explain the difference between a fixed effects and random effects model. 6 marks
  - c) Explain why *Blocking* is used in experimental design. Give an example. 5 marks
  - d) Consider a  $2^{7-4}$  fractional factorial design with the generators  $D = ABC$ ,  $E = AC$ ,  $F = BC$  and  $G = AB$ .
    - i) Write out the sign table (design matrix) for the factors A, B, C, D, E, F and G. 4 marks
    - ii) Write out the complete defining relation for this design. 6 marks
    - iii) What two Interactions are confounded with the main effect of Factor D? What Factor including two factor Interactions are confounded with the Interaction BG? 6 marks
    - iv) Identify the resolution of this design. Explain your answer. 2 marks

2.
  - a) An experiment was conducted to determine whether the type of plastic has any effect on the bursting strength. Four types of plastic were chosen. The experimenter was unable to obtain equal numbers of each type of plastic but decided to conduct the experiment nonetheless. The data presented is the bursting load.

Plastic Type			
A	B	C	D
23	19	11	22
25	20	13	21
24	18	12	23
	21	13	20
	22	14	

- i) Use one-way ANOVA to test for significant differences between the plastic types at the 1% level. Explain the meaning of your answer. 10 marks
  - ii) Is one compound significantly worse than the rest? Choose a significance level at which to evaluate. Explain your answer. 11 marks
- b). The accompanying data results from an experiment on bacterial growth.

No. of Days growth (x)	2	2.5	3	4	6
Bacteria growth (y)	15	22	40	100	1100

- i) By using the transformation  $y = ae^{bx}$ , show how linear regression techniques can be applied to the above data. 4 marks
  - ii) Determine the parameters of a suitable regression model and use this to predict the bacteria growth after 5 days. 8 marks

3.

- a) Explain how the *Inner Array* and the *Outer Array* are used in developing a robust product or process in Taguchi Methodology  
6 marks
- b) Explain the *Parameter Design* stage in Taguchi Methodology.  
5 marks
- c) An L4 Orthogonal Array was used to determine which of the factors A, B or C affect the diameter of a piston. The nominal diameter ( $y$ ) is 25.5mm. The purpose of the experiment was to achieve a nominal diameter of 25.5mm while also minimising the variation. The experiment is replicated three times and the results are presented below.

Run	A	B	C	$y_1$	$y_2$	$y_2$	$\bar{y}$	$s$
1	1	1	1	25	26	27		
2	1	2	2	20	30	40		
3	2	1	2	24	25	26		
4	2	2	1	25	45	35		

- i) Determine the appropriate Taguchi signal-to-noise ratio for each run.  
4 marks
- ii) By using the Taguchi signal-to-noise ratio, identify the Signal-to-Noise effects of the three factors on the response (i.e. diameter).  
4 marks
- iii) Using appropriate calculations, identify the factor that has the greatest effect on the nominal diameter.  
3 marks
- iv) Using the results from ii) and iii), determine the optimum process settings for manufacturing pistons. Explain your answer.  
3 marks
- v) Develop a model for the nominal diameter ( $y$ ) and predict the diameter using the optimum process settings determined in iv).  
3 marks
- vi) Develop a model for the predicted signal to noise ratio and use this to predict the standard deviation of the manufactured pistons at optimum process settings.  
5 marks

4.

An experiment is performed in a semiconductor fabrication plant in an effort to reduce the amount of waste produced. Five factors, each at two levels were studied. The experiment was replicated twice:

Variable	Low Level	High Level
A : Aperture Setting	Small	Large
B : Exposure Time (Secs)	10 sec	20 sec
C : Development Time	30 sec	45 sec
D : Mask Dimension	Small	Large
E : Etch time	14.5 min	15.5 min

A  $2^{5-2}$  fractional factorial design with design generator D = AB and E = AC and has been replicated twice.

The data on the waste produced at the various treatment combinations are shown in the following table.

Run	Replicate	
	I	II
<i>de</i>	34	30
<i>a</i>	36	42
<i>be</i>	12	18
<i>abd</i>	62	64
<i>cd</i>	24	18
<i>ace</i>	30	26
<i>bc</i>	34	28
<i>abcde</i>	56	60

An incomplete output from an ANOVA is presented on the following page. The magnitude and direction of the various effects are also included.

- From the data, complete the ANOVA table. 7 marks
- Determine which factors are statistically significant at the 1% level. 6 marks
- Develop the appropriate mathematical model for the response using the significant factor(s) and interaction(s). 4 marks
- Draw the response graph for the most significant factor and the most significant interaction. 5 marks
- What recommendations would you make regarding the design settings? What is the predicted response at these settings? 6 marks
- Explain the purpose of residual analysis. 5 marks

### Analysis of Variance for Waste Produced

Source	DF	SS	MS
A	?	?	?
B	1	552.25	552.25
C	1	30.25	30.25
D	1	930.25	930.25
E	1	110.25	110.25
B*C	1	272.25	272.25
B*E	1	110.25	110.25
Error	?	?	?
Total	?	?	

### Estimated Effects for Waste Produced

Term	Effect
A	?
B	11.750
C	-2.750
D	15.250
E	-5.250
B*C	8.250
B*E	-5.250

### Data for Question 4