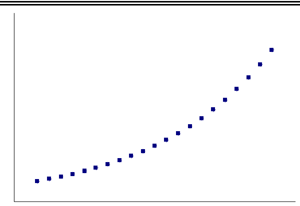
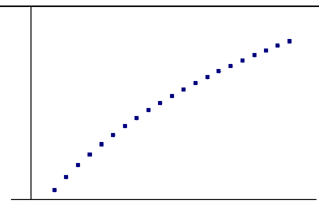
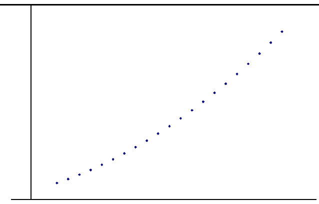
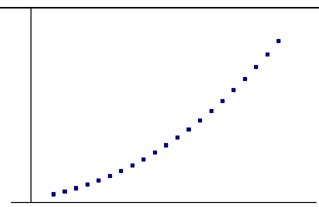
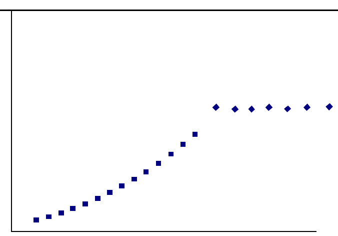
AP Statistics
Practice: Transformations to Achieve Linearity

Table of Common Transformations

There are many types of associations that you may encounter. This table lists the most common, and summarizes the way each is transformed. (Warning: This is by no means a complete list of every possible transformation!)

Appearance of Plot	Possible Association	Type of Transformation
	**Exponential*** General algebra equation: $y = a \times e^x$	Take the log (natural or base 10) of the dependent (y) variable.
	**Logarithmic** General algebra equation: $y = \ln x$	Take the log of the independent (x) variable.
	**Quadratic*** General algebra equation: $y = ax^2 + bx + c$	Take the square root of the dependent (y) variable.
	**Power*** General algebra equation: $y = a \times x^b$	Take the log of both variables.
	**Complex** More than one type of curve put together. Equations for different sections may vary.	partition data where the plot changes. Treat each part as a separate association.

*Note that these three associations have graphs that are very similar. The context of your data may give you a hint about the transformation to try first. You may need to try more than one to find the best transformation.

AP Statistics

Practice: Transformations to Achieve Linearity

General Procedure for Transforming Data

You must always start by looking at a scatterplot of your original data, and examining the pattern. Are there outliers or influential points? What is the shape of the curve? The shape is a guide to choosing a likely transformation. If the points seem to lie in a straight line, you may not need to transform the data at all; you may have a linear relationship.

If the explanatory variable involves time, particularly in years, you may want to change the variable to a form easier to work with. For example, if you were studying the U.S. population during the 1800's, looking for the effect of the Civil War, you might pick 1800 to be "zero." Then figure your explanatory variable in terms of years elapsed since 1800: 1820 becomes 20.

Remember to "Back-Transform" When you Predict *y*-Values

Remember that, whether you work with your calculator or a spreadsheet, you'll have results expressed simply in x and y. The correct variable for prediction is $\hat{y}$. And either variable may actually be transformed (ln, exponential, square, square root, and so on).

Things to Avoid

If your transformation involves taking a logarithm, remember that logarithms are undefined for zero and negative numbers.

Questions 1 through 6 work with the length of the sidereal year vs. distance from the sun. The table of data is shown below.

Planet	Distance from sun (in millions of miles)	Years (as a fraction of Earth years)	ln(Dist)	ln(Year)
Mercury	36.19	0.2410	3.5889	-1.4229
Venus	67.63	0.6156	4.2140	-0.4851
Earth	93.50	1.0007	4.5380	0.0007
Mars	142.46	1.8821	4.9591	0.6324
Jupiter	486.46	11.8704	6.1871	2.4741
Saturn	893.38	29.4580	6.7950	3.3830
Uranus	1,794.37	84.0100	7.4924	4.4309
Neptune	2,815.19	164.7800	7.9428	5.1046
pluto	3,695.95	248.5400	8.2150	5.5156

Enter the original data in L1 and L2 (that is, the Distance from the Sun and Years). Make L3 = ln(L1) and L4 = ln(L2). Verify that this matches the columns given above. Don't worry about the small discrepancies you may find due to rounding and the number of decimal places shown on your calculator. If your results differ from the values above, double-check your original entries!

AP Statistics
Practice: Transformations to Achieve Linearity

1. **Draw a scatterplot of Distance vs. Year (using the untransformed data) with the least-squares regression line. Does the line seem to model the relationship well?** (2 points)

2. On your calculator, do a linear regression (STAT **CALC** 8) for these different combinations:
   - Distance vs. ln(Year) (L1 vs. L4, if you entered the data as directed above)
   - Ln(Distance) vs. Year (L3 vs. L2, if you entered the data as directed above)
   - Ln(Distance) vs. Ln(Year) (L3 vs. L4, if you entered the data as directed above)

   (Note that the explanatory variable is always some form of "Distance.") To get the most out of this Assignment, look at a scatterplot of each of these combinations.

   Which transformation yields the highest correlation coefficient (Pearson's *r*)? sketch a scatterplot of this transformation and show the least-squares line. What is the value of *r* and r^2 for that transformation, and what regression equation does it yield? (3 points)

   (**Hint**: Remember to include "ln" on the variables in your regression equation that have been transformed.)

3. Using the regression equation from the previous question that *best fits the data*, place the values of the residuals into L5. In case you forgot how to do this:

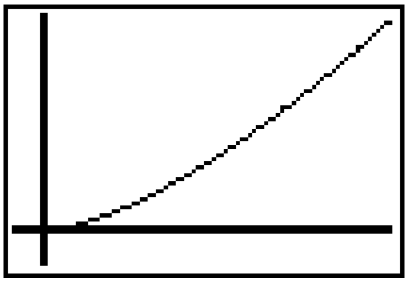
   press STAT 1 , highlight L5, in the data list window and press ENTER, then press 2nd [LIST], select **REsID**, and press ENTER ENTER.

   Create a residual plot on your calculator and interpret it; you don't need to draw the plot. (**Note:** You'll probably need to turn off the plot in Y1 to display the scatterplot correctly.) (2 points)

4. Using algebra, convert your regression e ation to a power equation (show your work below). Enter this equation in Y2 (press IY..=J and enter the equation) and make a scatterplot of LI, L2, with Y2, verifying that the power equation is a good fit for this data.

   As you set up your regression equation, keep in mind that the variables are Iny and Inx.

   Here's what the graph of the scatterplot and power equation will look like. (It's up to you to derive the power equation.)

   Finally, summarize, in plain English, what you've done in questions 1-4. (3 points)

5. The purpose of the transformations you're studying is to find a simple model to describe the relationship in a data set. The model can be used to predict a response value (called ***interpolation*** for values within the range of the data set and ***extrapolation*** for values outside the range of the data set). Recall that extrapolation is usually not a valid way to predict y-values.

   A well-known feature of our solar system is the asteroid belt between Mars and Jupiter. One theory about the asteroid belt is that it's made of primordial material that was prevented from forming another planet by the gravitational pull of Jupiter when the solar system was formed. One of the largest asteroids is 951 Gaspra. Its distance from the Sun is 207.16 million miles. Use your linear regression equation to interpolate the length of its sidereal year. (1 point)

   Remember that you need to take the natural log of Distance before you plug it in, and that your first result will be the natural log of Year. Show your work.

6. **Finally, calculate the length of the year for 951 Gaspra from the power function you developed in Question 4.** (Show all your work) (1 point)

   Note: Theoretically, the answers from 5 and 6 should be the same, but they'll probably come out differently due to rounding between steps. The more digits you carry throughout the calculations, the closer the two answers will be.

Questions 7 through 9 involve the following data set.

Increase in Life Expectancy in the United States during the 20th Century

Year	Expected Life Span	Year	Expected Life Span
1920	54.1	1975	72.6
1930	59.7	1980	73.7
1940	62.9	1985	74.7
1950	68.2	1990	75.4
1960	69.7	1995	75.8
1970	70.8		

Source: National Center for Health Statistics, published in the 1998 *World Almanac*

7. **Make a scatterplot of the *untransformed* data and tell which kind of relationship the points seem to follow. Also name the best type of transformation needed to "straighten" the plot.** (2 points)

   Note: Part of the transformation should involve subtracting 1900 from the year so you're working with more manageable numbers.

8. **Now try some transformations to get the data as close to linear as possible. (Use your calculator to transform the data, and try scatterplots of the different transformations). Then find the regression line, *r*, and r2.** (4 points)

   Tell which transformation worked the best and back it up by showing:
   - A scatterplot of the transformed data with the least-squares regression equation and line, r, and r-squared.
   - A plot of the original data with the regression equation converted to a non-linear equation (similar to what you did for question 6)
     (You'll get a chance to do a residual plot in the next question.)

   Here are some hints:
   - Look at the curve and think about what kind of relationship (equation) could have made such a curve (this is what you already did in question 7).
   - Then try the kind of transformation that should work for that kind of curve. (See the Summary of Common Transformations at the beginning of this document.) If your first guess doesn't work, try others.
   - More than one transformation will yield a good model; choose the one with the strongest value of *r*.
   - If your transformation (the one that seems to work the best) doesn't match your answer for question 7, you may want to revise your answer for question 7!

AP Statistics
Practice: Transformations to Achieve Linearity

Type of transformation:

Linear regression equation for transformed data:
rand $r2$:

Scatterplot of the transformed data with the least-squares regression equation and line, *r*, and $r2$.

Plot of the original data with the regression equation converted to a non-linear equation (similar to what you did for question 6).

9. **Using the transformed data and the regression equation for it, create a plot of residuals vs. x-values. Sketch the plot and interpret it.** (3 points)

10. The data below represents Medicare expenditures from 1970 to 1996, in billions of dollars. (4 points)

Year	Medicare Expenditures billions of dollars
1970	7.6
1980	37.5
1985	72.1
1990	112.1
1991	124.4
1992	141.4
1993	153
1994	169.8
1995	187.9
1996	203.1

For this data set, use your TI-83/TI-84 to:

A. **Create a scatterplot of the data.**

B. **Assume that the relationship of this data is exponential. Transform the data, find the regression equation, *r*, and r^2. Based only on the value of *r* would you consider this a good model for extrapolating increases in Medicare spending?**

C. **Create a residual plot of the transformed data. Does the residual plot change your mind about the usefulness of this model to extrapolate increases in Medicare spending? (Note:** Remember that many trends don't have a perfect mathematical model to predict them because there are too many complicating factors to yield a consistent curve. Sometimes in cases like these, a rough model will work as a rough estimator when used with appropriate caution.)