

OPRE 6332: Homework 4 (return before November 4, 2022, 23:59pm, on E-Learning)

You are the manager of a soccer team, and you need to decide which players to trade for the next season. Table 1 below describes the current players on your team that are up for contract renewal, listing their type, defense skill, passing skill, shooting skill, and market value. Any of these players can be sold to competing teams, and the revenue generated from such a transaction would be equal to their “market value.” For instance, if “Defender A” is sold, your team will receive \$1M.

Similarly, Table 2 lists players who are undergoing contract renewal at other teams. In other words, your team could acquire these players by paying their corresponding “market value.”

Player	Type	Defense Skill	Passing Skill	Shooting Skill	Market Value
A	Defender	9	5	2	1 million
B	Defender	7	6	3	0.5 million
C	Defender	6	6	4	0.6 million
D	Defender	7	7	1	1 million
E	Midfielder	8	9	5	3 million
F	Midfielder	4	8	6	1 million
G	Midfielder	6	7	7	1.8 million
H	Midfielder	2	7	6	1.2 million
I	Striker	5	7	10	8 million
J	Striker	2	5	8	1.5 million

Table 1. Players up for contract renewal on YOUR team.

Player	Type	Defense Skill	Passing Skill	Shooting Skill	Market Value
K	Defender	8	8	7	4 million
L	Defender	8	8	5	3 million
M	Midfielder	5	7	6	2.5 million
N	Midfielder	6	7	6	3 million
O	Striker	4	5	8	1.5 million

Table 2. Players available to acquire from other teams.

You are currently considering only trades that ensure the following conditions are met:

1. They would not require any additional capital. In other words, the funds generated from sold players would be sufficient to cover any acquisitions of new players.
2. You would not be changing the composition of your team roster. In other words, the total number of defenders, the total number of midfielders, and the total number of strikers would each remain constant, and match the totals in Table 1 (4 defenders, 4 midfielders, and 2 strikers). For example, selling one defender and acquiring a new defender would be valid, but selling one defender and acquiring a new striker would not.

3. The cumulative skill level of your new team should be at least as good as that of the old team, for each skill type (i.e., defense, passing, and shooting). Here, each skill level should be calculated by taking into account all the players in your team. For instance, the total defense skill of your current team is 56, so the defense skill of the new team should be at least 56.

Your primary goal is to maximize the total skill level of the new team. Here, total skill consists of the sum of skills (defense, passing, and shooting) across all players. For example, the total skill of your existing team is 175.

Questions

Note: If you feel that the case or one of the questions are ambiguous, state clearly any additional assumptions that you need to make, and proceed with the analysis accordingly.

1. **(50 points).** What is the maximal total skill that you can achieve in the new team, while satisfying all the desired conditions stated above (1,2, and 3)?
2. **(20 points).** Assume now that, if you don't make any changes to a particular section of your team (defense, midfield or strikers), then every player in that section will achieve a 5% increase in each type of skill. For instance, if you decide to not trade any of your midfielders, their ratings get a 5% boost, and are given by Table 3 below. Update your model in Question 1 to address this issue. What is the maximal skill level that can be achieved, and what trades should be made?

Clarification: for the purpose of this question, when interpreting condition (3), you should compare the skills of the new team with the existing skills of the old team (i.e., the skills of the old team without accounting for the 5% increase).

Player	Defense Skills	Passing Skills	Shooting Skills
Midfielder E	8.4	9.45	5.25
Midfielder F	4.2	8.4	6.3
Midfielder G	6.3	7.35	7.35
Midfielder H	2.1	7.35	6.3

Table 3. New skill levels for midfielders, if all midfielders are kept in the team.

3. **(20 points).** In addition to the change in Question 2, assume now that, if you maintain a section of your team unchanged, then each player in that section must receive a salary increase roughly equal to 25% of their market value. For instance, continuing with the example from Question 2, if you keep all your midfielders, then you must pay them an additional 25% of \$7M (in total). You would like these additional costs to come out of the funds generated from trading (i.e., you still do not want to inject any capital in the new season, just as in Question 1). Change the model in Question 2 to take this into account. How would your trades change?
4. **(10 points).** Consider the following hypothetical exercise. Suppose that, in addition to defense, passing, and shooting, your model had many more types of skills, and conditions such as (3) were enforced for every such skill. Suppose for a moment you were allowed to

ignore at most one such constraint. What changes would you make to your model in Question 1, so as to enable Solver to determine which constraint to ignore, as well as the optimal trades, so that total skill level is maximized in the new team. You do not need to implement anything in Excel here. A simple description of the new model suffices, clearly explaining what new decisions you would add, and how you would relate them (e.g., with constraints) to the existing ones.