

Exam 1 Take Home

You must submit ALL m-files via email to pandrist@greenriver.edu before class on Monday, May 30th. There must be a main script file named Exam1_*youremailaccount*.m and any function files you used in that script.

Root Solving Methods

1) Bisection

Water vapor splits into gaseous Hydrogen and Oxygen according to $2H_2O \rightarrow 2H_2 + O_2$. The mole fraction f of H_2O can be found from the equation

$$k = \frac{f}{1-f} \sqrt{\frac{2p_t}{2+f}}$$

where k is a reaction equilibrium constant and p_t is the total pressure of the mixture (in atmospheres). Solving this equation for f analytically is challenging, but because the mole fraction fundamentally varies from 0 to 1, a bracketed root finding method is well-suited to solve the problem.

Use a bisection method to solve for mole fraction f given $k = 0.04$ and p_t values varying from 0.25 atm to 10 atm. Solve the problem at many p_t values in that range and graph f vs. p_t (f should be on the vertical axis). Each data point in your graph will be built using the bisection method. You should use a loop to solve the bisection method many times for many different values of p_t .

2) Newton-Raphson

Write a Newton-Raphson code for the function to find a solution of $\cos(x) \sin(x) = 0.4$. Write the code twice, running with 2 different starting guesses.

- Choose a starting guess where the code returns an error or the method diverges (Please comment this out before submitting if it generates an error)
- Choose a starting guess where the method converges and a root is found

Eigenvalue problem – The popularity problem

The google search algorithm, in its infancy, solved a problem in linear algebra called the popularity problem. It attempts to answer the following question in a straight forward yet sophisticated way:

If I ask a group of n websites which among the other $n - 1$ sites are the most popular, how can I rank each website's popularity?

The simplest method is just count links. Every time a website is linked to, its popularity increases by 1. But this is problematic, and doesn't always give results that people agree with. Maybe one site is linked to many times, but it is exclusively from sites that are unpopular themselves. Also, maybe certain sites contain a ton of links, so their individual links shouldn't carry as much weight.

We would like to find the popularity of each website, where instead of simply counting links, we sum the popularity of each website that linked to each other website.

Read the following paragraph slowly and several times.

Let R_{ij} be a 1 or a 0 depending on whether or not the j^{th} website linked to the i^{th} website, and let p_i be the popularity of the i^{th} website. In that case, a sensible expression for the popularity of the 1st website would be

$$p_1 = R_{12}p_2 + \cdots R_{1n}p_n$$

So, for example, if website 1 was linked to by websites 3, 4, and 5, then website 1's popularity p_1 would be given by

$$p_1 = p_3 + p_4 + p_5$$

We could write a similar equation for every single website $i \in 1 \dots n$. This creates a system of equations of the form

$$\bar{R}\vec{p} = \vec{p}$$

where R is a matrix containing all of the R_{ij} entries, and \vec{p} is an unknown column vector containing the popularities of each website.

This should be immediately recognizable as an eigenvalue problem. Remember, the eigenvalue equation is $\bar{A}\vec{x} = \lambda\vec{x}$

It is not at all clear that the above equation ($\bar{R}\vec{p} = \vec{p}$) would, in general, have a solution. That's because it isn't a general eigenvalue equation. It specifically has an eigenvalue of 1. An arbitrary matrix \bar{R} will not necessarily have that eigenvalue. However, we can fix this problem if we enforce the following rule: every column of R must sum to 1. If every column adds up to one, the matrix is guaranteed to have 1 as an eigenvalue, and the above equation will have a solution. Essentially, each website is only allowed to give one total "link", evenly distributed amongst all the different websites it links to. The above fact is not obvious or simple to prove.

The j^{th} column of the R matrix, again, is the number of times the j^{th} website linked to the others. If we take each column and divide it by its sum, it will then meet our requirement. That is to say, we normalize each column, individually, so they all sum to 1. If the 5th website linked to you, but it also linked to 9 other websites, you only get an R value of 0.1. And that means that website 5's contribution to your popularity is not p_5 , but is only $0.1p_5$.

3) Ranking Websites

Using the method described above, rank the popularity of the following websites from most to least popular

	Website 1	Website 2	Website 3	Website 4	Website 5
Website 1	0	1	1	0	0
Website 2	1	0	1	0	1
Website 3	1	1	0	0	0
Website 4	1	1	1	0	0
Website 5	0	0	1	1	0

4) A Larger Matrix

Download data.mat from Canvas, and load the file using matlab. This will be a matrix in your workspace called R , which is another, much larger unnormalized R matrix. Sort the websites from most to least popular using the method described above. For sorting the popularities, use the sort function (built in to Matlab).

Newton's Method

5) Weird intersection

In this problem, you will be finding an intersection between a sphere, a paraboloid, and a plane. There are multiple solutions; you only need to find one.

$$\begin{aligned}x^2 + y^2 + z^2 &= 9 \\x^2 + y^2 - z &= 1 \\x + y + z &= 3\end{aligned}$$

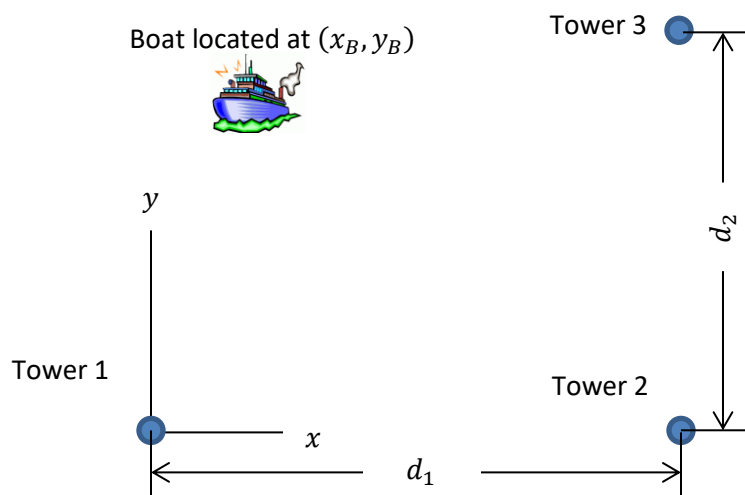
6) LORAN navigation (We did this in class, kinda)

The LORAN (LONG RANGE Navigation) system calculates the position of a boat at sea using signals from fixed transmitters. From the time difference of the incoming signals, it can determine its location at sea.

The system of equations generated is that of two hyperbolas, given below

$$\begin{aligned}\frac{4\left(x_B - \frac{d_1}{2}\right)^2}{c^2(t_2 - t_1)^2} - \frac{4y_B^2}{d_1^2 - c^2(t_2 - t_1)^2} &= 1 \\ \frac{4\left(y_B - \frac{d_2}{2}\right)^2}{c^2(t_3 - t_2)^2} - \frac{4(x_B - d_1)^2}{d_2^2 - c^2(t_3 - t_2)^2} &= 1\end{aligned}$$

Where c is the speed of light, and t_1, t_2 , and t_3 are the times at which the signals arrive from the 3 fixed transmitters. The descriptions of the other variables can be found in the figure below.



If tower 1 is 3 km from tower 2, and tower 3 is 4 km from tower 2, find the position of a boat at sea that receives signals at $t_1 = 0s$, $t_2 = 5.72 \mu s$, and $t_3 = 8.58 \mu s$

7) Orbital Mechanics

In orbital mechanics, the relationship between time and position for an elliptical orbit around the Earth is complicated! The position is tracked using an angle called the *true anomaly* (θ), and the orbit's size and shape are determined by parameters called the *semi-major axis* (a) and *eccentricity* (e). The process of solving for the amount of time (t) it takes to reach a particular true anomaly value for an orbit with a particular size and shape is complicated, and typically involved another intermediate quantity, called the *eccentric anomaly* (E). The 2 equations below are used to relate these quantities

$$\begin{aligned}t \sqrt{\frac{\mu}{a^3}} &= E - e \sin E \\(1 + e \cos \theta) \cos E &= e + \cos \theta\end{aligned}$$

For all Earth Orbits, $\mu = 3.986(10^5) \text{ km}^3/\text{s}^2$

For our orbit, $a = 25(10^3) \text{ km}$, $e = 0.5$

Use Newton's Method to build a code where, given t , you can find the true anomaly θ . Use this method for many different t values to build a graph of θ vs. t .