Lab 4: Independent Samples t-Tests

❖ When two samples are involved, the samples can come from different individuals who are not matched (the samples are independent of each other.)

❖ On the other hand, when the sample can come from the same individuals (the samples are paired with each other) and the samples are not independent of each other.

❖ The Independent Samples t-test can be used to see if two means are different from each other when the two samples were taken from different individuals who have not been matched.

The formula

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

follows the format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

The denominator is the ***standard error of the difference*** between two means.

❖ We will follow the steps:

1. Write the null (Ho) and alternative (H_1) hypotheses first:

H_0: $\mu_1 = \mu_2$	H_0: $\mu_1 = \mu_2$	H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$	H_1: $\mu_1 > \mu_2$	H_1: $\mu_1 < \mu_2$

Or

H_0: $\mu_1 - \mu_2 = 0$	H_0: $\mu_1 - \mu_2 = 0$	H_0: $\mu_1 - \mu_2 = 0$
H_1: $\mu_1 - \mu_2 \neq 0$	H_1: $\mu_1 - \mu_2 > 0$	H_1: $\mu_1 - \mu_2 < 0$

where μ_1 and μ_2 is the mean number of the sample1 and sample2, respectively.

2. Determine if this is a one-tailed or a two-tailed test and specify the α-level.

3. Use Megastat to find the p-value, t-value and F-value and other parameters

4. Check the test for equality of variances and select the appropriate t-value, accordingly.

5. Plot the t-distribution and show the critical and non-critical regions, locate the p-value and state the result whether the null hypothesis Ho is rejected or not.

6. Draw a conclusion: (choose one of the following situations)

   a. There is enough evidence to reject the claim that … if the claim is H_o

   b. There is no enough evidence to reject the claim that … if the claim is H_o

   c. There is enough evidence to support the claim that … if the claim is H_1

   d. There is no enough evidence to support the claim that … if the claim is H_1

Remarque: (on the p-value)

The p-value is the actual area under the t distribution curve and it is equal to the probability $\Pr(T \geq t)$; while α is the actual area under the t distribution curve and it is equal to the probability $\Pr(T \geq t_{vc})$.

We compare the p-value to the significance level α, whether it is a two-tailed or one-tailed test, such that:

• If the p-value $\leq \alpha$, reject the null hypothesis

• If the p-value $> \alpha$, do not reject the null hypothesis

Example: (**Too Long on the Telephone)**

A company collects data on the lengths of telephone calls made by employees in two different divisions. The sample mean and the sample standard deviation for the sales division are 10.26 and 8.56, respectively. The sample mean and sample standard deviation for the shipping and receiving division are 6.93 and 4.93, respectively. A hypothesis test was run, and the computer output follows.

Degrees of freedom = 56

Confidence interval limits = - 0.18979, 6.84979

Test statistic t = 1.89566

Critical value t = -2.0037, 2.0037

P-value = 0.06317

Significance level = 0.05

1. Are the samples independent or dependent?
2. Which number from the output is compared to the significance level to check if the null hypothesis should be rejected?
3. Which number from the output gives the probability of a type I error that is calculated from the sample data?
4. Was a right-, left-, or two-tailed test done? Why?
5. What are your conclusions?
6. What would your conclusions be if the level of significance were initially set at 0.10?

Solution:

1. These samples are independent.
2. We compare the P-value of 0.06317 to the significance level to check if the null hypothesis should be rejected.
3. The P-value of 0.06317 also gives the probability of a type I error.
4. Since two critical values are shown, we know that a two-tailed test was done.
5. Since the P-value of 0.06317 is greater than the significance value of 0.05, we fail to reject the null hypothesis and find that we do not have enough evidence to conclude that there is a difference in the lengths of telephone calls made by employees in the two divisions of the company.
6. If the significance level had been 0.10, we would have rejected the null hypothesis, since the P-value would have been less than the significance level.

Example 2: (Weights of Vacuum Cleaners)

Upright vacuum cleaners have either a hard body type or a soft body type. Shown are the weights in pounds of a random sample of each type. At $\alpha = 0.05$, can it be concluded that the means of the weights are different?

Hard body types				Soft body types			
21	17	17	20	24	13	11	13
16	17	15	20	12	15		
23	16	17	17				
13	15	16	18				
18							

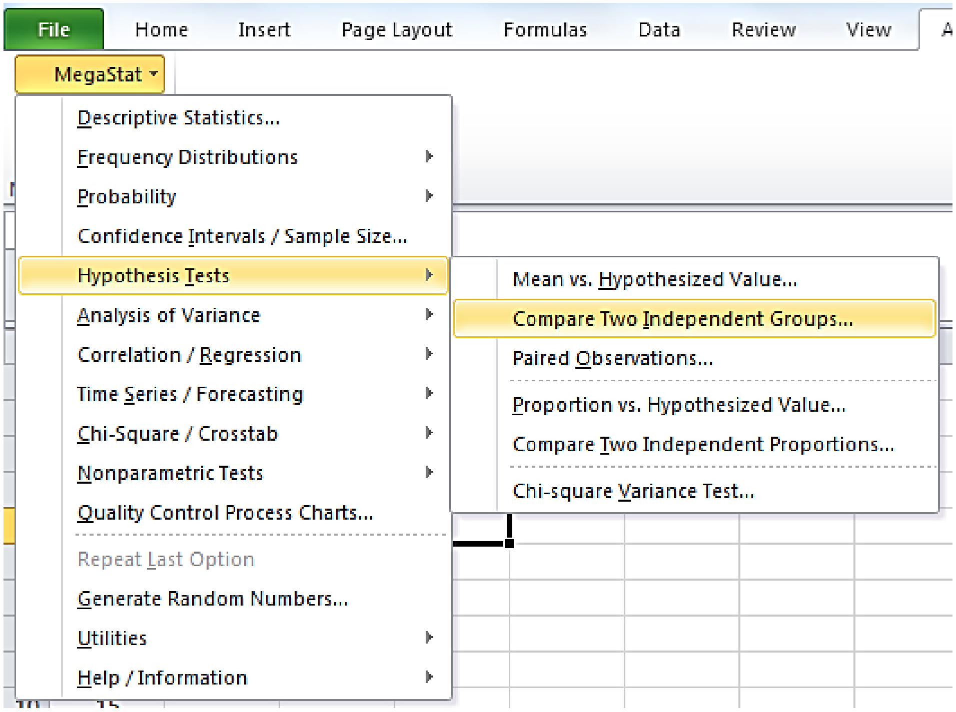
Solution:

Step 1: Group1= hard body type (Hard); Group2=soft body type (Soft)

$$\begin{cases} H_o : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \text{ (claim)} \end{cases}$$

Step 2: $\alpha = 0.05$, 2-tailed t-test

Step 3: Megastat calculation for T-test: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Frequency Distributions
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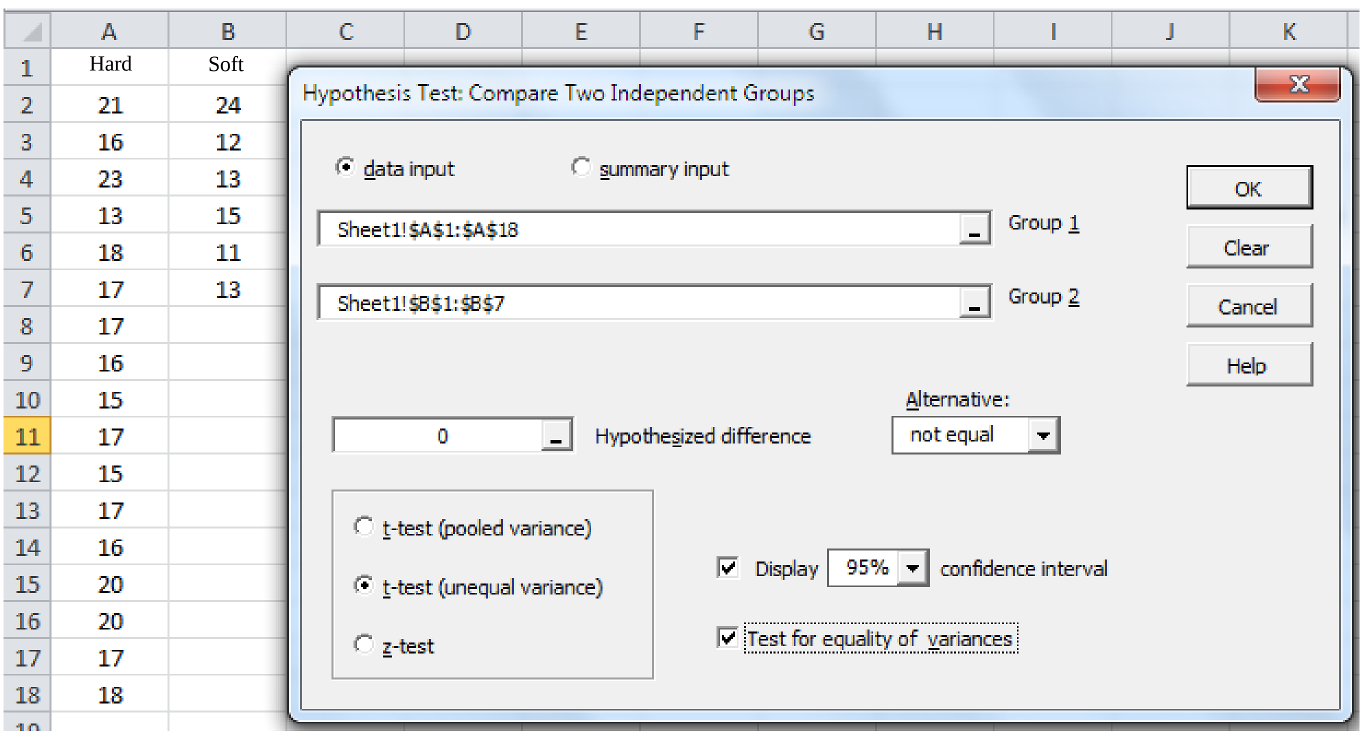
Hypothesis Tests
Analysis of Variance
Correlation / Regression
Time Series / Forecasting
Chi-Square / Crosstab
Nonparametric Tests
Quality Control Process Charts...
Repeat Last Option
Generate Random Numbers...
Utilities
Help / Information
Mean vs. Hypothesized Value...
Compare Two Independent Groups...
Paired Observations...
Proportion vs. Hypothesized Value...
Compare Two Independent Proportions...
Chi-square Variance Test...


A
B
C
D
E
F
G
H
I
J
K
Hard
Soft
21
24
16
12
23
13
13
15
18
11
17
13
17
16
15
17
15
17
16
20
20
17
18
Hypothesis Test: Compare Two Independent Groups
data input
summary input
Sheet1!$A$1:$A$18
Group 1
Sheet1!$B$1:$B$7
Group 2
OK
Clear
Cancel
Help
Alternative:
0
Hypothesized difference
not equal
t-test (pooled variance)
t-test (unequal variance)
z-test
Display
95%
confidence interval
Test for equality of variances

Hypothesis Test: Independent Groups (t-test, unequal variance)

Hard	Soft	
17.41	14.67	mean
2.45	4.76	std. dev.
17	6	n

5	df
2.745	difference (D - M)
2.033	standard error of difference
0	hypothesized difference
1.35	t
.2347	p-value (two-tailed)
-2.480	confidence interval 95.% lower
7.970	confidence interval 95.% upper
5.225	margin of error

F-test for equality of variance

22.67	variance: M
6.01	variance: D
3.77	F
.0380	p-value

Step 4: F-test for equality of variance shows the value of F = 3.77 associated with the p-value of 0.038.

F-test for equality of variance tell us whether an assumption of the t-test has been met (that is, the variances of the two groups are approximately equal). Look at the row labeled "p-value." under the *heading* " F-test for equality of variance ". There are two cases:

- If p-value. ≤ α, then reject the null hypothesis that the variance are equal (that is, the variances are not equal); thus choose the row of "*equal variances **not** assumed*".
- If p-value > α, then don't reject the null hypothesis that the variances are equal (that is, the variances are equal); thus choose in megastat the row of "*equal variances assumed*".

- In this example, at F = 0.173, p-value =0.682 which is greater than α. Therefore, the variances are equal and the row of "*equal variances assumed*" is considered.

Step 5: find the p-value of the t-test and make a decision: p-value = 0.365 which is greater than α. Thus, don't reject the null hypothesis

Step 6: one can conclude that there is not enough evidence to support the claim that the means of the weights are different.

Exercises:

Weights of Running Shoes The weights in ounces of a sample of running shoes for men and women are shown. Test the claim that the means are different. Use the P-value method with $\alpha = 0.05$.

Men		Women		
10.4	12.6	10.6	10.2	8.8
11.1	14.7	9.6	9.5	9.5
10.8	12.9	10.1	11.2	9.3
11.7	13.3	9.4	10.3	9.5
12.8	14.5	9.8	10.3	11

Gasoline Prices A random sample of monthly gasoline prices was taken from 2005 and from 2011. The samples are shown. Using $\alpha = 0.01$, can it be concluded that gasoline cost less in 2005? Use the P-value method.

2005	2.017	2.468	2.502	2.701	3.13	2.56	
2011	3.345	3.807	4.074	3.972	3.553	4.192	3.424

21. Random Numbers Two sets of 15 random integers from 1 to 100 were generated by a calculator. They are shown below. At the 0.10 level of significance, can it be concluded that the means differ? What would you expect? Why?

Set 1	80	43	60	41	16	39	29	12	12	13	54	24	9	46	25
Set 2	94	53	28	83	26	86	72	2	85	36	23	81	15	1	100

Tax-Exempt Properties A tax collector wishes to see if the mean values of the tax-exempt properties are different for two cities. The values of the tax-exempt properties for the two random samples are shown. The data are given in millions of dollars. At $\alpha = 0.05$, is there enough evidence to support the tax collector's claim that the means are different?

City A				City B			
113	22	14	8	82	11	5	15
25	23	23	30	295	50	12	9
44	11	19	7	12	68	81	2
31	19	5	2	20	16	4	5